The Generalization of Fermat's Last Theorem

By Leszek W. Guła
Lublin-POLAND

22 March 2016

Abstract—
Only one in the world proper proof of Fermat's Last Theorem (FLT) for $n = 4$. The proof of FLT for odd prime numbers n. The proof of Jeśmanowicz's Conjecture. The proof of Goldbach's Conjecture. The Beal's Conjecture 1&2.
MSC: Primary: 11A41, 11D41, 11D45, 11P32; Secondary: 11D61, 11D75, 11D85.

Keywords— Beal Conjecture, Common Prime Factor, Diophantine Equations, Diophantine Inequalities, Exponential Equations, Fermat Equation, Greatest Common Divisor, Newton Binomial Formula.

I. INTRODUCTION

The Fermat's Last Theorem is the famous theorem.
The Jeśmanowicz's Conjecture [1] concerns the Diophantus Equation.
The Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [7]
The Beal's Conjecture is a generalization of Fermat's Last Theorem. [6]

II. THE PROOF OF FERMAT'S LAST THEOREM

Theorem 1.

$$\{(2a+b)b: a \in [0,1,2,\dots] \wedge b \in [3,5,7,\dots]\} = \{9,15,21,25,27,33,35,39,45,49,\dots\} \Longrightarrow$$

$$\{3,5,7,\dots\} - \{9,15,21,25,27,33,35,39,45,49,\dots\} = \{3,5,7,11,13,17,19,23,29,31,\dots\} = \mathbb{P}. \ [2]$$

Theorem 2 (FLT). *For all* $n \in \{3,4,5,\dots\}$ *and for all* $A,B,C \in \{1,2,3,\dots\}$: $A^n + B^n \neq C^n$.

Proof. Suppose that for some $n \in \{3,4,5,\dots\}$ and for some co-prime $A,B,C \in \{1,2,3,\dots\}$: $A^n + B^n = C^n$.

Remark 1. These hypothesis $A^4 + B^4 = c^2, A^4 + B^4 = C^4$ **are absolutely different**, inasmuch as on the strenght of theTheorem 3 for $c = C^2$ we get $\frac{(u+v)^4+(u-v)^4}{2} = (u^2+v^2)^2 + (2uv)^2 = C^2 = (u^2+v^2)^2 \equiv 0$.

Fermat did not proved his own theorem for $n = 4$. [4]

For some relatively prime $u, v \in \{1,2,3,\dots\}$ such that $u - v$ is positive and odd:

$$\{(u^2+v^2)^2 - (2uv)^2 = A^2 \wedge [4(u^2+v^2)uv = B^2 \Rightarrow X^4 + Y^4 = z^2 < c^2 \ [4]] \wedge (u^2+v^2)^2 + (2uv)^2 = c\}.$$

Sufficient that we prove FLT for $n = 4$ and for $n \in \mathbb{P}$. [4] This is the remark 1.

For some $n \in \{3,4,5,\dots\}$ and for some $A,B,C \in \{1,2,3,\dots\}$:

$$(A + B \le C \wedge A^2 + B^2 \le C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} \le C^{n-1}) \Longrightarrow A^n + B^n < C^n.$$

Hence it must be $(A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1})$. [3]

Without loss for this proof we can assume that $\{A, (B - C) \in [1,3,5,\dots] \wedge (4 \nmid B \vee 4 \nmid C)\}$.

A. Proof For $n = 4$.

For some $A, C, v \in \{1,3,5,\dots\}$ and for some $B \in \{6,10,14,\dots\}$ such that A, C and B are co-prime:

$$\Big\{[A - (C - B) = 2v \wedge A^2 + B^2 > C^2 \wedge C - B + 2v = A \wedge C - A + 2v = B \wedge (C - B) + (C - A) + 2v = C \wedge (C - A + 2v)^4 = (C - A + A)^4 - A^4 \wedge (C - B + 2v)^4 = (C - B + B)^4 - B^4] \Longrightarrow \Big[(C-A)^2 2v + \frac{3}{2}(C-A)(2v)^2 + (2v)^3 + \frac{4v^4}{C-A} = (C-A)^2 A + \frac{3}{2}(C-A)A^2 + A^3 \wedge (C-B)^2 2v + \frac{3}{2}(C-B)(2v)^2 + (2v)^3 + \frac{4v^4}{C-B} = (C-B)^2 B + \frac{3}{2}(C-B)B^2 + B^3 \wedge 2v^2 > (C-A)(C-B)\Big]\Big\}.$$

Thus

$$\left\{\frac{4v^4}{C-A}, \frac{4v^4}{C-B} \in [1,3,5,\dots] \wedge 2v^2 > (C-A)(C-B)\right\}.$$

Hence – For some $c, d, v \in \{1,3,5,\dots\}$ such that c, d are co-prime:

$$(c^4 = C - B \wedge 4d^4 = C - A \wedge c^4 + 2v = A \wedge 4d^4 + 2v = B \wedge v^2 > 2c^4d^4) \Rightarrow v > \sqrt{2}(cd)^2.$$

Therefore – For some $c, d, e \in \{1,3,5,\dots\}$ such that c, d and e are co-prime: $cde = v$.

Further it mus be – For some $c, d, e, A \in \{1,3,5,\dots\}$ such that c, d and e are co-prime:

$$(2cde + 4d^4)^4 = [(4d^4 + A)^2]^2 - (A^2)^2 = [(4d^4 + A)^2 + A^2](2d^4 + A)8d^4 \Rightarrow$$

$$2(ce + 2d^3)^4 = [(4d^4 + A)^2]^2 - (A^2)^2 = [(4d^4 + A)^2 + A^2](2d^4 + A).$$

We assume that for some co-prime $z, w, x \in \{1,3,5,\dots\}$ and for some $y \in \{6,10,14,\dots\}$:

$$\{zw = ce + 2d^3 \wedge x + y = 2d^4 + A + 2d^4 \wedge x = 2d^4 + A \wedge y = 2d^4 \wedge\ 2(zw)^4 = [(x+y)^2 + (x-y)^2]x = 2(x^2 + y^2)x \wedge z^4w^4 = (x^2 + y^2)x \wedge (z^2)^2 = x^2 + y^2 \wedge w^4 = x\} \Rightarrow 4 \mid y,$$

which is inconsistent with $4 \nmid y$. [2]

This is the proof for $n = 4$.

B. Proof For $n \in \mathbb{P}$. In [2] we have the proof of FLT for $n \in \mathbb{P}$.

Theorem 3 (LWG Theorem). *For each* $g \in \{8,12,16,\dots\}$ *or for each* $g \in \{3,5,7,\dots\}$ *there exist finitely many pairs* (s, t) *of positive integers such that*:

$$g = \left(\frac{g + d^2}{2d}\right)^2 - \left(\frac{g - d^2}{2d}\right)^2 = s^2 - t^2 = (s+t)(s-t) = \frac{g}{d}(s-t) = \frac{g}{d}d = g,$$

where $d \mid g$ *and* $d < \sqrt{g}$ *and -* $d, \frac{g}{d} \in \{2,4,6,\dots\}$ *with even* g *or* $d \in \{1,3,5,\dots\}$ *with odd* g. [2]

Theorem 4. *For all* $x, u, v \in \{1,2,3,\dots\}$ *such that* $\boldsymbol{gcd}(u, v) = 1$ *and* $u - v \in \{1,3,5,\dots\}$:

$$\left\{(u+v)^x(u-v)^x = \left[\frac{(u+v)^x + (u-v)^x}{2}\right]^2 - \left[\frac{(u+v)^x - (u-v)^x}{2}\right]^2 \wedge (u^2 - v^2)^{2+x} + (2uv)^{2+x} = (u^2 - v^2)^2(u^2 - v^2)^x + (2uv)^2(2uv)^x < (u^2 - v^2)^2(u^2 + v^2)^x + (2uv)^2(u^2 + v^2)^x = (u^2 + v^2)^{2+x}\right\} \Rightarrow (u^2 - v^2, 2uv, u^2 + v^2).\ [2]$$

Theorem 5. *Let* u *and* v *be two relatively prime natural numbers such that* $u - v$ *is positive and odd. Then – For any primitive Pythagorean triple* (x, y, z) *there exists different and only one shared pair* (u, v), *whence it implies the primitive Diophantus triple* $(u^2 - v^2, 2uv, u^2 + v^2) = (x, y, z)$. [5], [2]

III. THE PROOF OF JEŚMANOWICZ'S CONJECTURE

Conjecture 1 (Jeśmanowicz Conjecture). For all $x, y, z, u, v \in \{1,2,3,\dots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5,\dots\}$: $(u^2 - v^2)^x + (2uv)^y \neq (u^2 + v^2)^z$.

Proof. Suppose that for some $x, y, z, u, v \in \{1,2,3,\dots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5,\dots\}$: $(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z$.

On the strength of the Theorems 3,4 and 5 we will have – For some $x, y, z, u, v \in \{1,2,3,\dots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5,\dots\}$:

$$[(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z \wedge u^2 - v^2 + 2uv > u^2 + v^2 \wedge (u^2 - v^2)^2 + (2uv)^2 = (u^2 + v^2)^2] \Rightarrow [(u^2 - v^2)^x = (u^2 - v^2)^2 \wedge (2uv)^y = (2uv)^2 \wedge (u^2 + v^2)^z = (u^2 + v^2)^2] \Rightarrow$$

$$(x, y, z) = (2,2,2),$$

which is inconsistent with $(x, y, z) \neq (2,2,2)$. [2]

This is the proof.

IV. THE PROOF OF GOLDBACH'S CONJECTURE

Conjecture 2 (Goldbach Conjecture). For all $Z \in \{6,8,10,\dots\}$ and for some $X,Y \in \mathbb{P}: Z = X+Y$.

Proof. The key of this proof are two common prime factors: 2 and 3.

$$\{\mathbf{18},24,\mathbf{30},36,\mathbf{42},48,\mathbf{54},60,\mathbf{66},72,\mathbf{78},84,\mathbf{90},96,\mathbf{102},108,\mathbf{114},\dots\} =$$

$$\{(3Z):(3Z)=(3X)+(3Y) \wedge 3X \le 3Y \wedge X,Y \in \mathbb{P} \wedge (3X),(3Y) \in [\mathbf{9},\mathbf{15},\mathbf{21},\mathbf{27},\mathbf{33},\mathbf{39},\mathbf{45},\mathbf{51},\dots]\} \Longrightarrow$$

$$\{6,8,10,12,14,16,18,20,22,\dots\} = \{Z:\ Z = X+Y \wedge X \le Y \wedge X,Y \in \mathbb{P}\}.$$ [2]

This is the proof.

V. THE BEAL'S CONJECTURE 1&2

Conjecture 3 (Beal Conjecture 1). For some $x,y,z \in \{3,4,5\dots\}$ and for some $A,B,C \in \{1,2,3,\dots\}$ such that A,B and C have the common prime factor $\boldsymbol{p} \ge 2$:

$$A^x + B^y = C^z.$$

The direct proof in [2] is the complete proof of Beal's Conjecture 1.

Beal Conjecture 2. For all $x,y,z \in \{3,4,5\dots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions $[A,B,C]$ in $\{1,2,3,\dots\}$.

Proof. Suppose that for some $x,y,z \in \{3,4,5,\dots\}$ the equation

$$A^x + B^y = C^z$$

has primitive solutions $[A,B,C]$ in $\{1,2,3,\dots\}$.

Then only one number out of any solution $[A,B,C]$ is even.

From the above it follows that

$$[(A+B>C \wedge A^2+B^2>C^2) \vee (A+B=C \wedge A^2+B^2<C^2) \vee (A+B<C \wedge A^2+B^2<C^2)].$$

Remark 2. Let $\boldsymbol{cpf}(\boldsymbol{p}4,\boldsymbol{p}5,\boldsymbol{p}3)=\boldsymbol{p}$, where $\boldsymbol{p}$ is the odd common prime factor with the numbers of the primitive solution $[4,5,3]$ of the equation $4+5=3^2 \equiv 1$.

If $2>1=1$, then for $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},5)=5$: $\boldsymbol{p}4+\boldsymbol{p}5=\boldsymbol{p}^2 3^2 \equiv 0$.

Hence the indirect proof of Beal Conjecture 2 in [2] is the false proof. This is the ramark 2.

REFERENCES

[1] Bobiński, Z., Kamiński, B.: WIADOMOŚCI MATEMATYCZNE XXXV, SERIES II, ROCZNIKI POLSKIEGO TOWARZYSTWA MATEMATYCZNEGO 1999 – http://main3.amu.edu.pl/~wiadmat/145-151_zb_wm35.pdf

[2] Guła, L. W.: Several Treasures of the Queen of Mathematics – http://www.ijetae.com/files/Volume6Issue1/IJETAE_0116_09.pdf

[3] Guła, L. W.: The Truly Marvellous Proof – http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf

[4] Narkiewicz, W.: WIADOMOŚCI MATEMATYCZNE XXX.1, Annuals PTM, Series II, Warszawa 1993.

[5] [4]. Husemöler, D.: Elliptic Curves, Second Edition, Springer, p. 7 http://www.math.rochester.edu/people/faculty/doug/otherpapers/Husemoller.pdf

[6] http://www.ams.org/profession/prizes-awards/ams-supported/beal-prize

[7] https://en.wikipedia.org/wiki/Goldbach%27s_conjecture