The Truly Marvellous Proofs

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***Abstract*—**
Two truly marvellous proofs of Fermat's Last Theorem (FLT). The truly marvellous proof of Jeśmanowicz's Conjecture (JC). Two truly marvellous proofs of Beal's Conjecture (BC). The truly marvellous proof of Goldbach's Conjecture (GC).
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I. INTRODUCTION

Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation the marginal comment that hints at the existence of a proof (a demonstratio sane mirabilis) of what has come to be known as Fermat's Last Theorem. [4] The proof 2 of FLT is dated July/August 1997.
Jeśmanowicz's Conjecture [1] concerns the Diophantus Equation.
The Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [7]
Beal's Conjecture is a generalization of Fermat's Last Theorem. [6]

II. TWO TRULY MARVELLOUS PROOFS OF FERMAT'S LAST THEOREM

Theorem 1 (LWG Theorem 1999).

$$\{(2a+b)b: a \in [0,1,2,\dots] \wedge b \in [3,5,7,\dots]\} = \{9,15,21,25,27,33,35,39,45,49,\dots\} \Longrightarrow$$

$$\{3,5,7,\dots\} - \{9,15,21,25,27,33,35,39,45,49,\dots\} = \{3,5,7,11,13,17,19,23,29,31,\dots\} = \mathbb{P}.\ [2]$$

Theorem 2 (FLT). *For all* $n \in \{3,4,5,\dots\}$ *and for all* $A,B,C \in \{1,2,3,\dots\}$: $A^n + B^n \neq C^n$.

Proof 1. Let for some $n \in \{3,4,5,\dots\}$ and for some co-prime $A,B,C \in \{1,2,3,\dots\}$: $A^n + B^n = C^n$.

Remark 1. Fermat did not proved his own theorem for $n = 4$. [5] Moreover

$$[(A^2)^2 + (B^2)^2 = (C^2)^2 \equiv 0 \vee (A^2)^2 + (B^2)^2 = c^2 \equiv 0]$$

because – For consecutive co-prime $u,v \in \{1,2,3,\dots\}$ such that $u - v \in \{1,3,5,\dots\}$:

$$\left[(u^2+v^2)^2 - (2uv)^2 = (u^2-v^2)^2 = A^2 \wedge uv(u^2+v^2)\sqrt{2} = B\right] \Longrightarrow B \notin \{4,6,8,\dots\}.\ [2]$$

Sufficient that we prove FLT for $n = 4$ and for $n \in \mathbb{P}$. [5] This is the remark 1.

For some $n \in \{3,4,5,\dots\}$ and for some $A,B,C \in \{1,2,3,\dots\}$:

$$(A + B \leq C \wedge A^2 + B^2 \leq C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} \leq C^{n-1}) \Longrightarrow A^n + B^n < C^n.$$

Hence it must be $(A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1})$. [3]

Without loss for this proof we can assume that $\{A, (B - C) \in [1,3,5,\dots] \wedge (4 \nmid B \vee 4 \nmid C)\}$.

A. Proof For $n = 4$.

For some $A,C,v \in \{1,3,5,\dots\}$ and for some $B \in \{6,10,14,\dots\}$ such that A, C and B are co-prime:

$$\Big\{[A - (C - B) = 2v \wedge A^2 + B^2 > C^2 \wedge C - B + 2v = A \wedge C - A + 2v = B \wedge (C - B) + (C - A) + 2v$$
$$= C \wedge (C - A + 2v)^4 = (C - A + A)^4 - A^4 \wedge (C - B + 2v)^4 = (C - B + B)^4 - B^4]$$
$$\Longrightarrow \Big[(C-A)^2 2v + \frac{3}{2}(C-A)(2v)^2 + (2v)^3 + \frac{4v^4}{C-A}$$
$$= (C-A)^2 A + \frac{3}{2}(C-A)A^2 + A^3 \wedge (C-B)^2 2v + \frac{3}{2}(C-B)(2v)^2 + (2v)^3 + \frac{4v^4}{C-B}$$
$$= (C-B)^2 B + \frac{3}{2}(C-B)B^2 + B^3 \wedge 2v^2 > (C-A)(C-B)\Big]\Big\}.$$

Thus

$$\left\{\frac{4v^4}{C-A},\frac{4v^4}{C-B}\in[1,3,5,\dots]\wedge 2v^2>(C-A)(C-B)\right\}.$$

Hence – For some $c,d,v\in\{1,3,5,\dots\}$ such that c,d are co-prime:

$$(c^4=C-B\wedge 4d^4=C-A\wedge 4d^4+2v=B\wedge v^2>2c^4d^4)\Longrightarrow v>\sqrt{2}(cd)^2.$$

Therefore – For some $c,d,e\in\{1,3,5,\dots\}$ such that c,d and e are co-prime: $cde=v$.

Further it mus be – For some $c,d,e,A\in\{1,3,5,\dots\}$ such that c,d and e are co-prime:

$$(2cde+4d^4)^4=[(4d^4+A)^2]^2-(A^2)^2=[(4d^4+A)^2+A^2](2d^4+A)8d^4\Longrightarrow$$
$$2(ce+2d^3)^4=[(4d^4+A)^2]^2-(A^2)^2=[(4d^4+A)^2+A^2](2d^4+A).$$

We assume that for some co-prime $z,w,x\in\{1,3,5,\dots\}$ and for some $y\in\{6,10,14,\dots\}$:

$$\{zw=ce+2d^3\wedge x+y=2d^4+A+2d^4\wedge x=2d^4+A\wedge y=2d^4\wedge\ 2(zw)^4=[(x+y)^2+(x-y)^2]x$$
$$=2(x^2+y^2)x\wedge z^4w^4=(x^2+y^2)x\wedge(z^2)^2=x^2+y^2\wedge w^4=x\}\Longrightarrow 4\mid y,$$

which is inconsistent with $4\nmid y$. [2] This is the proof for $n=4$.

B. Proof For $n\in\mathbb{P}$. In [2] we have the proof of FLT for $n\in\mathbb{P}$.

This is the proof 1 of FLT.

Proof 2. Let for some $n\in\{3,4,5,\dots\}$ and for some co-prime $A,B,C\in\{1,2,3,\dots\}$: $A^n+B^n=C^n$.

Then only one number out of any solution $[A,B,C]$ is even. Thus $A+B-C\in\{\dots,-6,-4,-2,0,2,4,6,\dots\}$.

Without loss for this proof we can assume that $A,(B-C)\in\{1,3,5,\dots\}$.

From FE it follows that for any solution $[A,B,C]$ and for all relatively prime $u,v\in\{0,1,2,\dots\}$ such that $u-v\in\{\dots,-5,-3,-1,1,3,5,\dots\}$:

$$\neg[A=u^2-v^2\wedge B=2uv\wedge -C=-(u^2+v^2)]\Longrightarrow\neg[A+B-C=2v(u-v)]\Longrightarrow$$
$$A+B-C\notin\{\dots,-6,-4,-2,0,2,4,6,\dots\},$$

which is inconsistent with $A+B-C\in\{\dots,-6,-4,-2,0,2,4,6,\dots\}$.

This is the proof 2 of FLT.

Theorem 3 (LWG Theorem 03&04 June1997). *For each* $g\in\{8,12,16,\dots\}$ *or for each* $g\in\{3,5,7,\dots\}$ *there exist finitely many pairs* (s,t) *of positive integers such that*:

$$g=\left(\frac{g+d^2}{2d}\right)^2-\left(\frac{g-d^2}{2d}\right)^2=s^2-t^2=(s+t)(s-t)=\frac{g}{d}(s-t)=\frac{g}{d}d=g,$$

where $d\mid g$ *and* $d<\sqrt{g}$ *and -* $d,\frac{g}{d}\in\{2,4,6,\dots\}$ *with even* g *or* $d\in\{1,3,5,\dots\}$ *with odd* g. [2]

Theorem 4. *For all* $x,u,v\in\{1,2,3,\dots\}$ *such that* $\boldsymbol{gcd}(u,v)=1$ *and* $u-v\in\{1,3,5,\dots\}$:

$$\left\{(u+v)^x(u-v)^x=\left[\frac{(u+v)^x+(u-v)^x}{2}\right]^2-\left[\frac{(u+v)^x-(u-v)^x}{2}\right]^2\wedge(u^2-v^2)^{2+x}+(2uv)^{2+x}\right.$$
$$=(u^2-v^2)^2(u^2-v^2)^x+(2uv)^2(2uv)^x<(u^2-v^2)^2(u^2+v^2)^x+(2uv)^2(u^2+v^2)^x$$
$$\left.=(u^2+v^2)^{2+x}\right\}\Longrightarrow(u^2-v^2,2uv,u^2+v^2).\ [2]$$

Theorem 5. *Let* u *and* v *be two relatively prime natural numbers such that* $u-v$ *is positive and odd. Then – For any primitive Pythagorean triple* (x,y,z) *there exists different and only one shared pair* (u,v), *which gives only one primitive Diophantus triple* $(u^2-v^2,2uv,u^2+v^2)=(x,y,z)$.

III. THE TRULY MARVELLOUS PROOF OF JEŚMANOWICZ'S CONJECTURE

Conjecture 1 (JC). For all $x,y,z,u,v\in\{1,2,3,\dots\}$ such that $(x,y,z)\neq(2,2,2)$ and $\boldsymbol{gcd}(u,v)=1$ and $u-v\in\{1,3,5,\dots\}$: $(u^2-v^2)^x+(2uv)^y\neq(u^2+v^2)^z$.

Proof. Suppose that for some $x,y,z,u,v \in \{1,2,3,...\}$ such that $(x,y,z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u,v) = 1$ and $u - v \in \{1,3,5,...\}$: $(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z$.

On the strength of the Theorems 3,4 and 5 we will have – For some $x,y,z,u,v \in \{1,2,3,...\}$ such that $(x,y,z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u,v) = 1$ and $u - v \in \{1,3,5,...\}$:

$$[(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z \wedge u^2 - v^2 + 2uv > u^2 + v^2 \wedge (u^2 - v^2)^2 + (2uv)^2 = (u^2 + v^2)^2]$$
$$\Longrightarrow [(u^2 - v^2)^x = (u^2 - v^2)^2 \wedge (2uv)^y = (2uv)^2 \wedge (u^2 + v^2)^z = (u^2 + v^2)^2] \Longrightarrow$$
$$(x,y,z) = (2,2,2),$$

which is inconsistent with $(x,y,z) \neq (2,2,2)$. [2]

This is the proof.

IV. Two Truly Marvellous Proofs Of Beal's Conjecture

Conjecture 2 (BC). For some $x,y,z \in \{3,4,5 ...\}$ and for some $A,B,C \in \{1,2,3,...\}$ such that A,B and C have the common prime factor $\boldsymbol{p} \geq 2$:

$$A^x + B^y = C^z.$$

The direct proof 1 in [2] is the complete proof of BC.

Or – For all $x,y,z \in \{3,4,5 ...\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions $[A,B,C]$ in $\{1,2,3,...\}$.

Proof 2. Suppose that for some $x,y,z \in \{3,4,5,...\}$ the equation

$$A^x + B^y = C^z$$

has primitive solutions $[A,B,C]$ in $\{1,2,3,...\}$.

Then only one number out of any solution $[A,B,C]$ is even. Thus $A + B - C \in \{...,-6,-4,-2,0,2,4,6,...\}$.

Without loss for this proof we can assume that $A,(B - C) \in \{1,3,5,...\}$.

On the strength of the proof of JC (also in [2]) it must be – For any solution $[A,B,C]$ and for all relatively prime $u,v \in \{0,1,2,...\}$ such that $u - v \in \{...,-5,-3,-1,1,3,5,...\}$:

$$\neg[A = u^2 - v^2 \wedge B = 2uv \wedge -C = -(u^2 + v^2)] \Longrightarrow \neg[A + B - C = 2v(u - v)] \Longrightarrow$$
$$A + B - C \notin \{...,-6,-4,-2,0,2,4,6,...\},$$

which is inconsistent with $A + B - C \in \{...,-6,-4,-2,0,2,4,6,...\}$.

This is the proof.

Remark 2. Let $\boldsymbol{cpf}(\boldsymbol{p}4,\boldsymbol{p}5,\boldsymbol{p}3) = \boldsymbol{p}$, where $\boldsymbol{p}$ is the odd common prime factor with the numbers of the primitive solution $[4,5,3]$ of the equation $4 + 5 = 3^2 \equiv 1$.

If $2 > 1 = 1$, then for $\boldsymbol{p} = \boldsymbol{cpf}(\boldsymbol{p},5) = 5$: $\boldsymbol{p}4 + \boldsymbol{p}5 = \boldsymbol{p}^2 3^2 \equiv 0$.

Hence the indirect proof of BC in [2] is the false proof. This is the ramark 2.

V. The Truly Marvellous Proof Of Goldbach's Conjecture

Conjecture 3 (GC). For all $Z \in \{6,8,10,...\}$ and for some $X,Y \in \mathbb{P}$: $Z = X + Y$.

Proof. The key of this proof are two common prime factors: 2 and 3.

$$\{\mathbf{18},24,\mathbf{30},36,\mathbf{42},48,\mathbf{54},60,\mathbf{66},72,\mathbf{78},84,\mathbf{90},96,\mathbf{102},108,\mathbf{114},...\} =$$
$$\{(3Z)\colon\ (3Z) = (3X) + (3Y) \wedge 3X \leq 3Y \wedge X,Y \in \mathbb{P} \wedge (3X),(3Y) \in [\mathbf{9},\mathbf{15},\mathbf{21},\mathbf{27},\mathbf{33},\mathbf{39},\mathbf{45},\mathbf{51},...]\} \Longrightarrow$$
$$\{6,8,10,12,14,16,18,20,22,...\} = \{Z\colon\ Z = X + Y \wedge X \leq Y \wedge X,Y \in \mathbb{P}\}.\ [2]$$

This is the proof.

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