The Generalization of Fermat's Last Theorem

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01 March 2016

Abstract—
The proof of Fermat's Last Theorem (FLT). The proof of Jeśmanowicz's Conjecture. The proof of Goldbach's Conjecture. The Beal's Conjecture 1&2.

MSC: Primary: 11A41, 11D41, 11D45, 11P32; Secondary: 11D61, 11D75, 11D85.

Keywords— Beal Conjecture, Common Prime Factor, Diophantine Equations, Diophantine Inequalities, Exponential Equations, Fermat Equation, Greatest Common Divisor, Newton Binomial Formula.

I. INTRODUCTION

Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation the marginal comment that hints at the existence of a proof (a demonstratio sane mirabilis) of what has come to be known as Fermat's Last Theorem. [4]
Jeśmanowicz's Conjecture [1] concerns the Diophantus Equation.
The Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [7]
Beal's Conjecture is a generalization of Fermat's Last Theorem. [6]

II. THE PROOF OF FERMAT'S LAST THEOREM

Theorem 1 (LWG Theorem 1999).

$$\{(2a+b)b : a \in [0,1,2,\dots] \wedge b \in [3,5,7,\dots]\} = \{9,15,21,25,27,33,35,39,45,49,\dots\} \Longrightarrow$$

$$\{3,5,7,\dots\} - \{9,15,21,25,27,33,35,39,45,49,\dots\} = \{3,5,7,11,13,17,19,23,29,31,\dots\} = \mathbb{P}. \text{ [2]}$$

Theorem 2 (FLT). *For all* $n \in \{3,4,5,\dots\}$ *and for all* $A,B,C \in \{1,2,3,\dots\}$: $A^n + B^n \neq C^n$.

Proof. Suppose that for some $n \in \{3,4,5,\dots\}$ and for some co-prime $A,B,C \in \{1,2,3,\dots\}$: $A^n + B^n = C^n$.

Remark 1. Fermat did not proved his own theorem for $n = 4$. [5] Moreover

$$[(A^2)^2 + (B^2)^2 = (C^2)^2 \equiv 0 \vee (A^2)^2 + (B^2)^2 = c^2 \equiv 0],$$

inasmuch as then – For some $u, v \in \{1,2,3,\dots\}$ such that $\boldsymbol{gcd}(u,v) = 1$ and $u - v \in \{1,3,5,\dots\}$:

$$\left[(u^2+v^2)^2 - (2uv)^2 = (u^2-v^2)^2 = A^2 \wedge uv(u^2+v^2)\sqrt{2} = B\right] \Longrightarrow B \notin \{4,6,8,\dots\}. \text{ [2]}$$

Sufficient that we prove FLT for $n = 4$ and for $n \in \mathbb{P}$. [5] This is the remark 1.

For some $n \in \{3,4,5,\dots\}$ and for some $A,B,C \in \{1,2,3,\dots\}$:

$$(A + B \leq C \wedge A^2 + B^2 \leq C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} \leq C^{n-1}) \Longrightarrow A^n + B^n < C^n.$$

Hence it must be $(A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1})$. [3]

Without loss for this proof we can assume that $\{A, (B - C) \in [1,3,5,\dots] \wedge (4 \nmid B \vee 4 \nmid C)\}$.

A. Proof For $n = 4$.

For some $A, C, v \in \{1,3,5,\dots\}$ and for some $B \in \{6,10,14,\dots\}$ such that A, C and B are co-prime:

$$\Big\{[A - (C - B) = 2v \wedge A^2 + B^2 > C^2 \wedge C - B + 2v = A \wedge C - A + 2v = B \wedge (C - B) + (C - A) + 2v$$
$$= C \wedge (C - A + 2v)^4 = (C - A + A)^4 - A^4 \wedge (C - B + 2v)^4 = (C - B + B)^4 - B^4]$$
$$\Longrightarrow \Big[(C - A)^2 2v + \frac{3}{2}(C - A)(2v)^2 + (2v)^3 + \frac{4v^4}{C - A}$$
$$= (C - A)^2 A + \frac{3}{2}(C - A)A^2 + A^3 \wedge (C - B)^2 2v + \frac{3}{2}(C - B)(2v)^2 + (2v)^3 + \frac{4v^4}{C - B}$$
$$= (C - B)^2 B + \frac{3}{2}(C - B)B^2 + B^3 \wedge 2v^2 > (C - A)(C - B)\Big]\Big\}.$$

Thus

$$\left\{\frac{4v^4}{C-A},\frac{4v^4}{C-B}\in[1,3,5,\dots]\wedge 2v^2>(C-A)(C-B)\right\}.$$

Hence – For some $c,d,v\in\{1,3,5,\dots\}$ such that c,d are co-prime:

$$(c^4=C-B\wedge 4d^4=C-A\wedge 4d^4+2v=B\wedge v^2>2c^4d^4)\Rightarrow v>\sqrt{2}(cd)^2.$$

Therefore – For some $c,d,e\in\{1,3,5,\dots\}$ such that c,d and e are co-prime: $cde=v$.

Further it mus be – For some $c,d,e,A\in\{1,3,5,\dots\}$ such that c,d and e are co-prime:

$$(2cde+4d^4)^4=[(4d^4+A)^2]^2-(A^2)^2=[(4d^4+A)^2+A^2](2d^4+A)8d^4\Rightarrow$$
$$2(ce+2d^3)^4=[(4d^4+A)^2]^2-(A^2)^2=[(4d^4+A)^2+A^2](2d^4+A).$$

We assume that for some co-prime $z,w,x\in\{1,3,5,\dots\}$ and for some $y\in\{6,10,14,\dots\}$:

$$\{zw=ce+2d^3\wedge x+y=2d^4+A+2d^4\wedge x=2d^4+A\wedge y=2d^4\wedge\ 2(zw)^4=[(x+y)^2+(x-y)^2]x$$
$$=2(x^2+y^2)x\wedge z^4w^4=(x^2+y^2)x\wedge(z^2)^2=x^2+y^2\wedge w^4=x\}\Rightarrow 4\mid y,$$

which is inconsistent with $4\nmid y$. [2]

This is the proof for $n=4$.

B. Proof For $n\in\mathbb{P}$. In [2] we have the proof of FLT for $n\in\mathbb{P}$.

Theorem 3 (LWG Theorem 03&04 June1997). *For each $g\in\{8,12,16,\dots\}$ or for each $g\in\{3,5,7,\dots\}$ there exist finitely many pairs (s,t) of positive integers such that*:

$$g=\left(\frac{g+d^2}{2d}\right)^2-\left(\frac{g-d^2}{2d}\right)^2=s^2-t^2=(s+t)(s-t)=\frac{g}{d}(s-t)=\frac{g}{d}d=g,$$

where $d\mid g$ and $d<\sqrt{g}$ and - $d,\frac{g}{d}\in\{2,4,6,\dots\}$ with even g or $d\in\{1,3,5,\dots\}$ with odd g. [2]

Theorem 4. *For all $x,u,v\in\{1,2,3,\dots\}$ such that $\boldsymbol{gcd}(u,v)=1$ and $u-v\in\{1,3,5,\dots\}$*:

$$\Bigg\{(u+v)^x(u-v)^x=\left[\frac{(u+v)^x+(u-v)^x}{2}\right]^2-\left[\frac{(u+v)^x-(u-v)^x}{2}\right]^2\wedge(u^2-v^2)^{2+x}+(2uv)^{2+x}$$
$$=(u^2-v^2)^2(u^2-v^2)^x+(2uv)^2(2uv)^x<(u^2-v^2)^2(u^2+v^2)^x+(2uv)^2(u^2+v^2)^x$$
$$=(u^2+v^2)^{2+x}\Bigg\}\Rightarrow(u^2-v^2,2uv,u^2+v^2).\ [2]$$

Theorem 5. *For any primitive Diophantus triple (Pythagorean triple) $(u^2-v^2,2uv,u^2+v^2)$ there exists different and only one shared pair (u,v).* [2]

III. THE PROOF OF JEŚMANOWICZ'S CONJECTURE

Conjecture 1 (Jeśmanowicz Conjecture). For all $x,y,z,u,v\in\{1,2,3,\dots\}$ such that $(x,y,z)\neq(2,2,2)$ and $\boldsymbol{gcd}(u,v)=1$ and $u-v\in\{1,3,5,\dots\}$: $(u^2-v^2)^x+(2uv)^y\neq(u^2+v^2)^z$.

Proof. Suppose that for some $x,y,z,u,v\in\{1,2,3,\dots\}$ such that $(x,y,z)\neq(2,2,2)$ and $\boldsymbol{gcd}(u,v)=1$ and $u-v\in\{1,3,5,\dots\}$: $(u^2-v^2)^x+(2uv)^y=(u^2+v^2)^z$.

On the strength of the Theorems 3,4 and 5 we will have – For some $x,y,z,u,v\in\{1,2,3,\dots\}$ such that $(x,y,z)\neq(2,2,2)$ and $\boldsymbol{gcd}(u,v)=1$ and $u-v\in\{1,3,5,\dots\}$:

$$[(u^2-v^2)^x+(2uv)^y=(u^2+v^2)^z\wedge u^2-v^2+2uv>u^2+v^2\wedge(u^2-v^2)^2+(2uv)^2=(u^2+v^2)^2]$$
$$\Rightarrow[(u^2-v^2)^x=(u^2-v^2)^2\wedge(2uv)^y=(2uv)^2\wedge(u^2+v^2)^z=(u^2+v^2)^2]\Rightarrow$$
$$(x,y,z)=(2,2,2),$$

which is inconsistent with $(x,y,z)\neq(2,2,2)$. [2]

This is the proof.

IV. THE PROOF OF GOLDBACH'S CONJECTURE

Conjecture 2 (Goldbach Conjecture). For all $Z \in \{6,8,10,\dots\}$ and for some $X,Y \in \mathbb{P}: Z = X + Y$.

Proof. The key of this proof are two common prime factors: 2 and 3.

$$\{\mathbf{18}, 24, \mathbf{30}, 36, \mathbf{42}, 48, \mathbf{54}, 60, \mathbf{66}, 72, \mathbf{78}, 84, \mathbf{90}, 96, \mathbf{102}, 108, \mathbf{114}, \dots\} =$$

$$\{(3Z){:}\ (3Z) = (3X) + (3Y) \wedge 3X \leq 3Y \wedge X,Y \in \mathbb{P} \wedge (3X),(3Y) \in [\mathbf{9}, \mathbf{15}, \mathbf{21}, \mathbf{27}, \mathbf{33}, \mathbf{39}, \mathbf{45}, \mathbf{51}, \dots]\} \Longrightarrow$$

$$\{6,8,10,12,14,16,18,20,22,\dots\} = \{Z{:}\ Z = X + Y \wedge X \leq Y \wedge X,Y \in \mathbb{P}\}.\ [2]$$

It is easy to verify that

$$[18 = 9 + 9, 24 = 9 + 15, 30 = 9 + 21 = 15 + 15, 36 = 15 + 21, 42 = 9 + 33 = 21 + 21, 48 = 9 + 39 = 15 + 33, 54 = 15 + 39 = 21 + 33, 60 = 9 + 51 = 21 + 39, 66 = 9 + 57 = 15 + 51 = 33 + 33, 72 = 15 + 57 = 21 + 51 = 33 + 39, 78 = 9 + 69 = 21 + 57 = 39 + 39, \dots].$$

Thus we get all equations according to the Goldbach's Conjecture, namely

$$[6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7 = 5 + 5, 12 = 5 + 7, 14 = 3 + 11 = 7 + 7, 16 = 3 + 13 = 5 + 11, 18 = 5 + 13 = 7 + 11, 20 = 3 + 17 = 7 + 13, 22 = 3 + 19 = 5 + 17 = 11 + 11, 24 = 5 + 19 = 7 + 17 = 11 + 13, 26 = 3 + 23 = 7 + 19 = 13 + 13, 28 = 5 + 23 = 11 + 17, 30 = 7 + 23 = 11 + 19 = 13 + 17, 32 = 3 + 29 = 13 + 19, \dots].$$

This is the proof.

V. THE BEAL'S CONJECTURE 1&2

Conjecture 3 (Beal Conjecture 1). For some $x,y,z \in \{3,4,5\dots\}$ and for some $A,B,C \in \{1,2,3,\dots\}$ such that A,B and C have the common prime factor $\boldsymbol{p} \geq 2$:

$$A^x + B^y = C^z.$$

The direct proof in [2] is the complete proof of Beal's Conjectuer 1.

Beal Conjecture 2. For all $x,y,z \in \{3,4,5\dots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions $[A,B,C]$ in $\{1,2,3,\dots\}$.

Proof. Suppose that for some $x,y,z \in \{3,4,5,\dots\}$ the equation

$$A^x + B^y = C^z$$

has primitive solutions $[A,B,C]$ in $\{1,2,3,\dots\}$.

Then only one number out of any solution $[A,B,C]$ is even.

From the above it follows that

$$[(A + B > C \wedge A^2 + B^2 > C^2) \vee (A + B = C \wedge A^2 + B^2 < C^2) \vee (A + B < C \wedge A^2 + B^2 < C^2)].$$

Remark 2. Let $\boldsymbol{cpf}(\boldsymbol{p}4, \boldsymbol{p}5, \boldsymbol{p}3) = \boldsymbol{p}$, where $\boldsymbol{p}$ is the odd common prime factor with the numbers of the primitive solution $[4,5,3]$ of the equation $4 + 5 = 3^2 \equiv 1$.

If $2 > 1 = 1$, then for $\boldsymbol{p} = \boldsymbol{cpf}(\boldsymbol{p}, 5) = 5$: $\boldsymbol{p}4 + \boldsymbol{p}5 = \boldsymbol{p}^2 3^2 \equiv 0$.

Hence the indirect proof of Beal Conjecture 2 in [2] is the false proof. This is the ramark 2.

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