The Generalization of Fermat's Last Theorem

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***Abstract*—**
The hypothetical Fermat's proof of Fermat's Last Theorem (FLT).
The proof of Jeśmanowicz's Conjecture.
The Beal's Conjecture 2.

MSC: Primary: 11A41, 11D41, 11D45; Secondary: 11D61, 11D75, 11D85.

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I. INTRODUCTION

Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation the marginal comment that hints at the existence of a proof (a demonstratio sane mirabilis) of what has come to be known as Fermat's Last Theorem. [4]
Jeśmanowicz's Conjecture [1] concerns the Diophantus Equation.
Beal's Conjecture is a generalization of Fermat's Last Theorem. [5]

II. THE HYPOTHETICAL FERMAT'S PROOF OF FERMAT'S LAST THEOREM

Theorem 1 (FLT). *For all* $n, A, B, C \in \{3,4,5,\dots\}$: $A^n + B^n \neq C^n$.

Proof. Suppose that for some $n \in \{3,4,5,\dots\}$ and for some co-prime $A, B, C \in \{1,2,3,\dots\}$:

Then $(A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1})$.

Then only one number out of any solution $[A, B, C]$ is even and $A, (B - C) \in \{1,3,5,\dots\}$. [2]

From the above it follows that – For some $A, B, C \in \{3,4,5,\dots\}$ and for all $v \in \{1,2,3,\dots\}$:

$$(A > 2v + 1\ [3]\ \wedge B \neq 2v^2 + 2v \wedge C \neq 2v^2 + 2v + 1\ [2]\ ) \Rightarrow A > 2v + 1 \equiv 0.$$

This is the proof.

Remark 1. It is known that $(n \mid A \veebar n \mid B \veebar n \mid C)$. [2] If $n \nmid ABC$, then a proof of FLT is false.

For some $v \in \{1,2,3,\dots\}$:

$$[A = C - B + 2v \wedge B = C - A + 2v \wedge C = C - B + C - A + 2v \wedge 2v^2 > (C - A)(C - B) \wedge n \mid v].$$

If $C = 2v^2 + 2v + 1$, then $n \nmid C$. If $B = 2v^2 + 2v$, then $n \nmid B$. If $A = 2v + 1$, then $n \nmid A$.

This is the remark 1.

Theorem 2 (LWG Theorem 03&04 June1997). *For each* $g \in \{8,12,16,\dots\}$ *or for each* $g \in \{3,5,7,\dots\}$ *there exist finitely many pairs* (s,t) *of positive integers such that*:

$$g = \left(\frac{g + d^2}{2d}\right)^2 - \left(\frac{g - d^2}{2d}\right)^2 = s^2 - t^2 = (s + t)(s - t) = \frac{g}{d}(s - t) = \frac{g}{d}d = g,$$

where $d \mid g$ *and* $d < \sqrt{g}$ *and* - $d, \frac{g}{d} \in \{2,4,6,\dots\}$ *with even* g *or* $d \in \{1,3,5,\dots\}$ *with odd* g. [2]

Theorem 3. *For all* $x, u, v \in \{1,2,3,\dots\}$ *such that* $\boldsymbol{gcd}(u,v) = 1$ *and* $u - v \in \{1,3,5,\dots\}$:

$$\left\{ (u + v)^x (u - v)^x = \left[\frac{(u + v)^x + (u - v)^x}{2}\right]^2 - \left[\frac{(u + v)^x - (u - v)^x}{2}\right]^2 \wedge (u^2 - v^2)^{2+x} + (2uv)^{2+x} \right.$$
$$= (u^2 - v^2)^2 (u^2 - v^2)^x + (2uv)^2 (2uv)^x < (u^2 - v^2)^2 (u^2 + v^2)^x + (2uv)^2 (u^2 + v^2)^x$$
$$\left. = (u^2 + v^2)^{2+x} \right\} \Rightarrow (u^2 - v^2, 2uv, u^2 + v^2).\ [2]$$

Theorem 4. *For any primitive Diophantus triple (Pythagorean triple)* $(u^2 - v^2, 2uv, u^2 + v^2)$ *there exists different and only one shared relatively prime pair* (u,v). [2]

III. THE PROOF OF JEŚMANOWICZ'S CONJECTURE

Conjecture 1 (Jeśmanowicz Conjecture). For all $x, y, z, u, v \in \{1,2,3, \ldots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5, \ldots\}$: $(u^2 - v^2)^x + (2uv)^y \neq (u^2 + v^2)^z$.

Proof. Suppose that for some $x, y, z, u, v \in \{1,2,3, \ldots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5, \ldots\}$: $(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z$.

On the strength of the Theorems 2,3, and 4 we will have – For some $x, y, z \in \{1,2,3, \ldots\}$ and for some $u, v \in \{1,2,3, \ldots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5, \ldots\}$:

$$[(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z \wedge u^2 - v^2 + 2uv > u^2 + v^2 \wedge (u^2 - v^2)^2 + (2uv)^2 = (u^2 + v^2)^2] \\ \Longrightarrow [(u^2 - v^2)^x = (u^2 - v^2)^2 \wedge (2uv)^y = (2uv)^2 \wedge (u^2 + v^2)^z = (u^2 + v^2)^2] \Longrightarrow (x, y, z) \\ = (2,2,2),$$

which is inconsistent with $(x, y, z) \neq (2,2,2)$. [2]

This is the proof.

IV. THE BEAL'S CONJECTURE 2

Conjecture 2 (Beal Conjecture 2). For all $x, y, z \in \{3,4,5 \ldots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions $[A, B, C]$ in $\{1,2,3, \ldots\}$.

Proof. Let for some $x, y, z \in \{3,4,5, \ldots\}$ and for some co-prime $A, B, C \in \{1,2,3, \ldots\}$: $A^x + B^y = C^z$.

Then only one number out of any solution $[A, B, C]$ is even.

From the above it follows that

$$[(A^x + B^y = C^z \wedge A + B > C \wedge A^2 + B^2 > C^2) \vee (A^x + B^y = C^z \wedge A + B = C \wedge A^2 + B^2 < C^2) \\ \vee (A^x + B^y = C^z \wedge A + B < C \wedge A^2 + B^2 < C^2)].$$

Remark 2. Let $\boldsymbol{cpf}(\boldsymbol{p}4, \boldsymbol{p}5, \boldsymbol{p}3) = \boldsymbol{p}$, where $\boldsymbol{p}$ is the odd common prime factor with the numbers of the primitive solution $[4,5,3]$ of the equation $4 + 5 = 3^2$.

If $2 > 1 = 1$, then for $\boldsymbol{p} = \boldsymbol{cpf}(\boldsymbol{p}, 5) = 5$:

$$\boldsymbol{p}4 + \boldsymbol{p}5 = \boldsymbol{p}^2 3^2 \Longrightarrow 4 + 5 = \boldsymbol{p}3^2 \Longrightarrow \boldsymbol{cpf}(\boldsymbol{p}, 4) > 1,$$

which is inconsistent with primitive solution $[4,5,3]$? Therefore these proofs in [2] are false proofs.

This is the ramark 2.

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