

The Generalization of Fermat's Last Theorem

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Abstract—

The hypothetical Fermat's proof of Fermat's Last Theorem (FLT).

The proof of Jeśmanowicz's Conjecture.

The Beal's Conjecture 2.

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I. INTRODUCTION

Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation the marginal comment that hints at the existence of a proof (a demonstratio sane mirabilis) of what has come to be known as Fermat's Last Theorem. [4]

Jeśmanowicz's Conjecture [1] concerns the Diophantus Equation.

Beal's Conjecture is a generalization of Fermat's Last Theorem. [5]

II. THE HYPOTHETICAL FERMAT'S PROOF OF FERMAT'S LAST THEOREM

Theorem 1 (FLT). For all $n \in \{3,4,5, \dots\}$ and for all co-prime $A, B, C \in \{1,2,3, \dots\}$: $A^n + B^n \neq C^n$.

Proof. Suppose that for some $n \in \{3,4,5, \dots\}$ and for some co-prime $A, B, C \in \{1,2,3, \dots\}$:

$$[A^{n+1} + B^{n+1} < C^{n+1} \wedge A^n + B^n = C^n \wedge (A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1})]. \quad [2]$$

Only one number out of any solution $[A, B, C]$ is even and $A + B > C$, so $A, C - B \in \{1,3,5, \dots\}$ and –

For some co-prime $u, v \in \{1,2,3, \dots\}$ such that $u - v \in \{1,3,5, \dots\}$: $A + B - C = 2v(u - v)$. We have

$$[A = C - B + 2v(u - v) \wedge B = C - A + 2v(u - v) \wedge C = C - B + C - A + 2v(u - v) \wedge A^2 + B^2 > C^2].$$

Thus $2v^2(u - v)^2 > (C - A)(C - B)$, with $[(n | v \wedge n | A) \vee (n | v \wedge n | B) \vee (n | v \wedge n | C)]$.

If $C = u^2 + v^2$, then $n \nmid C$, inasmuch as $\mathbf{gcd}(u, v) = 1$. Then the proof is incomplete.

If $[A = u^2 - v^2 \vee A = (u - v)^n + 2v(u - v)]$, then $n \nmid A$ [3] and the proof is incomplete.

If $B = 2uw$, then $(A = u^2 - a - v^2 \wedge C = u^2 - a + v^2 \wedge n | v \wedge n \nmid C, A)$ and the proof is incomplete.

Hence – For any solution $[A, B, C]$ and for some co-prime $u, v \in \{1,2,3, \dots\}$ such that $u - v \in \{1,3,5, \dots\}$:

$$[A \neq u^2 - v^2 \wedge B \neq 2uw \wedge C \neq u^2 + v^2 \wedge A + B - C = 2v(u - v)] \Rightarrow A \notin \{1,3,5, \dots\},$$

which is inconsistent with $A, C - B \in \{1,3,5, \dots\}$. This is the proof.

Theorem 2 (LWG Theorem 03&04 June1997). For each $g \in \{8,12,16, \dots\}$ or for each $g \in \{3,5,7, \dots\}$ there exist finitely many pairs (s, t) of positive integers such that:

$$g = \left(\frac{g + d^2}{2d}\right)^2 - \left(\frac{g - d^2}{2d}\right)^2 = s^2 - t^2 = (s + t)(s - t) = \frac{g}{d}(s - t) = \frac{g}{d}d = g,$$

where $d|g$ and $d < \sqrt{g}$ and $-d, \frac{g}{d} \in \{2,4,6, \dots\}$ with even g or $d \in \{1,3,5, \dots\}$ with odd g . [2]

Theorem 3. For all $x, u, v \in \{1,2,3, \dots\}$ such that $\mathbf{gcd}(u, v) = 1$ and $u - v \in \{1,3,5, \dots\}$:

$$\left\{ \begin{aligned} (u + v)^x (u - v)^x &= \left[\frac{(u + v)^x + (u - v)^x}{2} \right]^2 - \left[\frac{(u + v)^x - (u - v)^x}{2} \right]^2 \wedge (u^2 - v^2)^{2+x} + (2uw)^{2+x} \\ &= (u^2 - v^2)^2 (u^2 - v^2)^x + (2uw)^2 (2uw)^x < (u^2 - v^2)^2 (u^2 + v^2)^x + (2uw)^2 (u^2 + v^2)^x \\ &= (u^2 + v^2)^{2+x} \end{aligned} \right\} \Rightarrow (u^2 - v^2, 2uw, u^2 + v^2). \quad [2]$$

Theorem 4. For any primitive Diophantus triple (Pythagorean triple) $(u^2 - v^2, 2uv, u^2 + v^2)$ there exists different and only one shared pair (u, v) . [2]

III. THE PROOF OF JEŚMANOWICZ'S CONJECTURE

Conjecture 1 (Jeśmanowicz Conjecture). For all $x, y, z, u, v \in \{1, 2, 3, \dots\}$ such that $(x, y, z) \neq (2, 2, 2)$ and $\gcd(u, v) = 1$ and $u - v \in \{1, 3, 5, \dots\}$: $(u^2 - v^2)^x + (2uv)^y \neq (u^2 + v^2)^z$.

Proof. Suppose that for some $x, y, z, u, v \in \{1, 2, 3, \dots\}$ such that $(x, y, z) \neq (2, 2, 2)$ and $\gcd(u, v) = 1$ and $u - v \in \{1, 3, 5, \dots\}$: $(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z$.

On the strength of the Theorems 2, 3, and 4 we will have – For some $x, y, z \in \{1, 2, 3, \dots\}$ and for some $u, v \in \{1, 2, 3, \dots\}$ such that $(x, y, z) \neq (2, 2, 2)$ and $\gcd(u, v) = 1$ and $u - v \in \{1, 3, 5, \dots\}$:

$$\begin{aligned} [(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z \wedge u^2 - v^2 + 2uv > u^2 + v^2 \wedge (u^2 - v^2)^2 + (2uv)^2 = (u^2 + v^2)^2] \\ \Rightarrow [(u^2 - v^2)^x = (u^2 - v^2)^2 \wedge (2uv)^y = (2uv)^2 \wedge (u^2 + v^2)^z = (u^2 + v^2)^2] \Rightarrow (x, y, z) \\ = (2, 2, 2), \end{aligned}$$

which is inconsistent with $(x, y, z) \neq (2, 2, 2)$. [2] This is the proof.

IV. THE BEAL'S CONJECTURE 2

Conjecture 2 (Beal Conjecture 2). For all $x, y, z \in \{3, 4, 5, \dots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions $[A, B, C]$ in $\{1, 2, 3, \dots\}$.

Proof. Let for some $x, y, z \in \{3, 4, 5, \dots\}$ and for some coprime $A, B, C \in \{1, 2, 3, \dots\}$: $A^x + B^y = C^z$.

Then only one number out of any solution $[A, B, C]$ is even.

From the above it follows that

$$[(A^x + B^y = C^z \wedge A + B > C \wedge A^2 + B^2 > C^2) \vee (A^x + B^y = C^z \wedge A + B \leq C \wedge A^2 + B^2 < C^2)].$$

Remark 1. Let $\text{cpf}(p4, p5, p3) = p$, where p is the odd common prime factor with the numbers of the primitive solution $[4, 5, 3]$ of the equation $4 + 5 = 3^2$.

If $2 > 1 = 1$, then for $p = \text{cpf}(p, 5) = 5$:

$$p4 + p5 = p^2 3^2 \Rightarrow 4 + 5 = p3^2 \Rightarrow \text{cpf}(p, 4) > 1,$$

which is inconsistent with primitive solution $[4, 5, 3]$? Therefore these proofs in [2] are false proofs.

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