Three Wonderful Proofs

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***Abstract*—**
The wonderful proof of Fermat’s Last Theorem.
The wonderful proof of Jeśmanowicz’s Conjecture.

The wonderful proof of Beal's Conjecture 2.

MSC: Primary: 11A41, 11D41, 11D45; Secondary: 11D61, 11D75, 11D85.

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I. INTRODUCTION

The Fermat’s Last Theorem (FLT) is the famous theorem. The proof of FLT is dated July/August 1997.
Jeśmanowicz’s Conjecture [1] concerns the Diophantus Equation.
Beal’s Conjecture is a generalization of Fermat’s Last Theorem. [5]

II. THE WONDERFUL PROOF OF FERMAT’S LAST THEOREM

Theorem 1 (Femat Last Theorem). *For all* $n \in \{3,4,5,\dots\}$ *and for all* $A,B,C \in \{1,2,3,\dots\}$:

$$A^n + B^n \neq C^n.$$

Proof. Suppose that for some $n \in \{3,4,5,\dots\}$ and for some $A,B,C \in \{1,2,3,\dots\}$:

$$(A^n + B^n = C^n \wedge A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1}).$$

In the another case we will have – For some $n \in \{3,4,5,\dots\}$ and for some $A,B,C \in \{1,2,3,\dots\}$:

$$(A^n + B^n = C^n \wedge A + B \le C \wedge A^2 + B^2 \le C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} \le C^{n-1}) \Rightarrow A^n + B^n < C^n,$$

which is inconsistent with $A^n + B^n = C^n$. [2]

We assume that A, B and C are co-prime. Then only one number out of each solution $[A,B,C]$ is even.

Thus $A + B - C \in [2,4,6,\dots]$. Without loss for this proof we can assume that $A, C - B \in \{1,3,5,\dots\}$.

We assume – For some co-prime $u,v \in \{1,2,3,\dots\}$ such that $u - v \in \{1,3,5,\dots\}$: $A + B - C = 2v(u-v)$.

$[A = C - B + 2v(u-v) \wedge B = C - A + 2v(u-v) \wedge C = C - B + C - A + 2v(u-v) \wedge A^2 + B^2 > C^2]$.

Thus $2v^2(u-v)^2 > (C-A)(C-B)$, with $[(n \mid v \wedge n \mid A) \vee (n \mid v \wedge n \mid B) \vee (n \mid v \wedge n \mid C)]$. [2]

If $A = u^2 - v^2$, then $n \nmid A$. Then the proof is incomplete.

If $C = u^2 + v^2$, then $n \nmid C$. Then the proof is incomplete.

If $B = 2uv$, then $(A = u^2 + a - v^2 \wedge C = u^2 + a + v^2 \wedge n \mid v \wedge n \nmid C, A)$. Then the proof is incomplete.

If $A = (u-v)^n + 2v(u-v)$, then $n \nmid A$. [3] Then the proof is incomplete.

From the Fermat Equation it follows that – For any primitive solution $[A,B,C]$ and for all $u,v \in \{1,2,3,\dots\}$ such that $\boldsymbol{gcd}(u,v) = 1$ and $u - v \in \{1,3,5,\dots\}$:

$$\{\neg[(A = u^2 - v^2 \wedge B = 2uv \wedge C = u^2 + v^2) \equiv 0 \Longrightarrow A + B - C = 2v(u-v) \equiv 1]$$
$$\equiv 0 \wedge \neg[(A \neq u^2 - v^2 \wedge B \neq 2uv \wedge C \neq u^2 + v^2) \equiv 1 \Longrightarrow A + B - C \neq 2v(u-v) \equiv 0]$$
$$\equiv 1\} \equiv 0.$$

Equally from the Fermat Equation it follows that – For some co-prime $A,B,C \in \{1,2,3,\dots\}$ and for some $u,v \in \{1,2,3,\dots\}$ such that $\boldsymbol{gcd}(u,v) = 1$ and $u - v \in \{1,3,5,\dots\}$:

$$\{[(A = u^2 - v^2 \wedge B = 2uv \wedge C = u^2 + v^2) \equiv 0 \Longrightarrow A + B - C = 2v(u-v) \equiv 1]$$
$$\equiv 1 \vee [(A \neq u^2 - v^2 \wedge B \neq 2uv \wedge C \neq u^2 + v^2) \equiv 1 \Longrightarrow A + B - C \neq 2v(u-v) \equiv 0] \equiv 0\}$$
$$\equiv 1.$$

Therefore $\{[(\neg \boldsymbol{T}) \Rightarrow \boldsymbol{q}] \wedge [(\neg \boldsymbol{T}) \Rightarrow (\neg \boldsymbol{q})]\} \Longrightarrow \boldsymbol{T}$. [4] This is the wonderful proof.

Theorem 2 (LWG Theorem 03&04 June1997). *For each* $g \in \{8,12,16,\dots\}$ *or for each* $g \in \{3,5,7,\dots\}$ *there exist finitely many pairs* (s,t) *of positive integers such that*:

$$g = \left(\frac{g+d^2}{2d}\right)^2 - \left(\frac{g-d^2}{2d}\right)^2 = s^2 - t^2 = (s+t)(s-t) = \frac{g}{d}(s-t) = \frac{g}{d}d = g,$$

where $d|g$ *and* $d < \sqrt{g}$ *and* - $d, \frac{g}{d} \in \{2,4,6,\dots\}$ *with even* g *or* $d \in \{1,3,5,\dots\}$ *with odd* g. [2]

Theorem 3. *For all* $x,u,v \in \{1,2,3,\dots\}$ *such that* $\boldsymbol{gcd}(u,v) = 1$ *and* $u - v \in \{1,3,5,\dots\}$:

$$\Bigg\{(u+v)^x(u-v)^x = \left[\frac{(u+v)^x + (u-v)^x}{2}\right]^2 - \left[\frac{(u+v)^x - (u-v)^x}{2}\right]^2 \wedge (u^2-v^2)^{2+x} + (2uv)^{2+x}$$
$$= (u^2-v^2)^2(u^2-v^2)^x + (2uv)^2(2uv)^x < (u^2-v^2)^2(u^2+v^2)^x + (2uv)^2(u^2+v^2)^x$$
$$= (u^2+v^2)^{2+x}\Bigg\}. \ [2]$$

Definition 1. $\boldsymbol{cpf}[\boldsymbol{p}(u^2-v^2), \boldsymbol{p}2uv, \boldsymbol{p}(u^2+v^2)] = \boldsymbol{p}$, where $\boldsymbol{p}$ is the odd common prime factor $(\boldsymbol{cpf})$ with the numbers of the solutions $[u^2-v^2, 2uv, u^2+v^2]$ such that $\boldsymbol{p}, uv, u^2-v^2$ are co-prime. [2]

This is the definition 1.

Definition 2. $\boldsymbol{cpf}(\boldsymbol{p}A, \boldsymbol{p}B, \boldsymbol{p}C) = \boldsymbol{p}$, where $\boldsymbol{p}$ is the odd common prime factor $(\boldsymbol{cpf})$ with the numbers of the primitive solutions $[A,B,C]$. [2]

This is the definition 2.

III. THE WONDERFUL PROOF OF JEŚMANOWICZ'S CONJECTURE

Conjecture 1 (Jeśmanowicz Conjecture). For all $x,y,z,u,v \in \{1,2,3,\dots\}$ such that $(x,y,z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u,v) = 1$ and $u - v \in \{1,3,5,\dots\}$: $(u^2-v^2)^x + (2uv)^y \neq (u^2+v^2)^z$.

Proof. Suppose that for some $x,y,z,u,v \in \{1,2,3,\dots\}$ such that $(x,y,z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u,v) = 1$ and $u - v \in \{1,3,5,\dots\}$: $(u^2-v^2)^x + (2uv)^y = (u^2+v^2)^z$.

If $u = 2$ and $v = 1$, then

$$(3 + 4 > 5 \wedge 3^1 + 4^2 < 5^2 \wedge 3^2 + 4^1 < 5^2 \wedge 3^1 + 4^3 > 5^2 \wedge 3^3 + 4^1 > 5^2 \wedge 3^3 + 4^2 < 5^3 \wedge 3^2 + 4^3 < 5^3).$$

If $u - v > v$, then $u^2 - v^2 + 2uv > u^2 + v^2$ and

$$[(u^2-v^2)^1 + (2uv)^2 < (u^2+v^2)^2 \wedge (u^2-v^2)^2 + (2uv)^1 < (u^2+v^2)^2] \wedge$$
$$[(u^2-v^2)^1 + (2uv)^3 > (u^2+v^2)^2 \wedge (u^2-v^2)^3 + (2uv)^1 > (u^2+v^2)^2] \wedge$$
$$[(u^2-v^2)^3 + (2uv)^2 < (u^2+v^2)^3 \wedge (u^2-v^2)^2 + (2uv)^3 < (u^2+v^2)^3].$$

If $u - v < v$, then $u^2 - v^2 + 2uv > u^2 + v^2$ and

$$[(u^2-v^2)^1 + (2uv)^2 < (u^2+v^2)^2 \wedge (u^2-v^2)^2 + (2uv)^1 < (u^2+v^2)^2] \wedge$$
$$[(u^2-v^2)^1 + (2uv)^3 > (u^2+v^2)^2 \wedge (u^2-v^2)^3 + (2uv)^1 < (u^2+v^2)^2] \wedge$$
$$[(u^2-v^2)^3 + (2uv)^2 < (u^2+v^2)^3 \wedge (u^2-v^2)^2 + (2uv)^3 < (u^2+v^2)^3].$$

On the strength of the Theorem 3 – For some $z \in \{3,4,5,\dots\}$ and for some $p,q \in \{0,1,2,\dots\}$ and for some $u,v \in \{1,2,3,\dots\}$ such that $p + q > 0$ and $u - v \in \{1,3,5,\dots\}$ and $\boldsymbol{p}, uv, u - v$ are co-prime:

$$[(u^2-v^2)^{z+p} + (2uv)^{z-q} = (u^2+v^2)^z \vee (u^2-v^2)^{z-p} + (2uv)^{z+q} = (u^2+v^2)^z].$$

Then for some $\boldsymbol{p}$ it must be $\boldsymbol{p} = \boldsymbol{cpf}(\boldsymbol{p}, u^2+v^2)$.

If $z + p > z \geq z - q$, then for some $\boldsymbol{p} = \boldsymbol{cpf}(\boldsymbol{p}, u^2+v^2)$:

$$[\boldsymbol{p}^{z+p}(u^2-v^2)^{z+p} + \boldsymbol{p}^{z-q}(2uv)^{z-q} = \boldsymbol{p}^z(u^2+v^2)^z] \Longrightarrow$$
$$\{[\boldsymbol{p}^{p+q}(u^2-v^2)^{z+p} + (2uv)^{z-q} = \boldsymbol{p}^q(u^2+v^2)^z \vee \boldsymbol{p}^p(u^2-v^2)^{z+p} + (2uv)^z = (u^2+v^2)^z]\}$$
$$\Longrightarrow [\boldsymbol{cpf}(\boldsymbol{p}, (2uv)^{z-q}) > 1 \vee \boldsymbol{cpf}(\boldsymbol{p}, (2uv)^z) > 1],$$

which is inconsistent with $\boldsymbol{p}, uv, u^2 - v^2$ are co-prime.

If $z+q>z\geq z-p$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},u^2+v^2)$:

$$[\boldsymbol{p}^{z-p}(u^2-v^2)^{z-p}+\boldsymbol{p}^{z+q}(2uv)^{z+q}=\boldsymbol{p}^{z}(u^2+v^2)^{z}]\Rightarrow$$

$$\{[(u^2-v^2)^{z-p}+\boldsymbol{p}^{p+q}(2uv)^{z+q}=\boldsymbol{p}^{p}(u^2+v^2)^{z}\vee(u^2-v^2)^{z-p}+\boldsymbol{p}^{q}(2uv)^{z+q}=(u^2+v^2)^{z}]\}$$
$$\Rightarrow[\boldsymbol{cpf}(\boldsymbol{p},(u^2-v^2)^{z-p})>1\vee\boldsymbol{cpf}(\boldsymbol{p},(u^2-v^2)^{z-p})>1],$$

which is inconsistent with $\boldsymbol{p},uv,u^2-v^2$ are co-prime. This is the wonderful proof.

IV. THE WONDERFUL PROOF OF BEAL'S CONJECTURE

Conjecture 2 (Beal Conjecture in the case 2). For all $x,y,z\in\{3,4,5\ldots\}$ the equation

$$A^x+B^y=C^z$$

has no primitive solutions $[A,B,C]$ in $\{1,2,3,\ldots\}$.

Proof. Let for some $x,y,z\in\{3,4,5,\ldots\}$ and for some coprime $A,B,C\in\{1,2,3,\ldots\}$: $A^x+B^y=C^z$.

Then only one number out of each solution $[A,B,C]$ is even. Thus we assume that A is odd.

If $y>x=z$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},A)$:

$$(\boldsymbol{p}^xA^x+\boldsymbol{p}^yB^y=\boldsymbol{p}^zC^z)\Rightarrow(A^x+\boldsymbol{p}^{y-z}B^y=C^z)\Rightarrow\boldsymbol{cpf}(\boldsymbol{p},C^z)>1,$$

which is inconsistent with the primitive solution $[A,B,C]$.

If $y>x>z$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},A)$:

$$(\boldsymbol{p}^xA^x+\boldsymbol{p}^yB^y=\boldsymbol{p}^zC^z)\Rightarrow(\boldsymbol{p}^{x-z}A^x+\boldsymbol{p}^{y-z}B^y=C^z)\Rightarrow\boldsymbol{cpf}(\boldsymbol{p},C^z)>1,$$

which is inconsistent with the primitive solution $[A,B,C]$.

If $y>z>x$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},C)$:

$$(\boldsymbol{p}^xA^x+\boldsymbol{p}^yB^y=\boldsymbol{p}^zC^z)\Rightarrow(A^x+\boldsymbol{p}^{y-x}B^y=\boldsymbol{p}^{z-x}C^z)\Rightarrow\boldsymbol{cpf}(\boldsymbol{p},A^z)>1,$$

which is inconsistent with the primitive solution $[A,B,C]$.

If $y=x>z$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},A)$:

$$(\boldsymbol{p}^xA^x+\boldsymbol{p}^yB^y=\boldsymbol{p}^zC^z)\Rightarrow(\boldsymbol{p}^{x-z}A^x+\boldsymbol{p}^{y-z}B^y=C^z)\Rightarrow\boldsymbol{cpf}(\boldsymbol{p},C^z)>1,$$

which is inconsistent with the primitive solution $[A,B,C]$.

If $z>y=x$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},A)$:

$$(\boldsymbol{p}^xA^x+\boldsymbol{p}^yB^y=\boldsymbol{p}^zC^z)\Rightarrow(A^x+B^y=\boldsymbol{p}^{z-x}C^z)\Rightarrow\boldsymbol{cpf}(\boldsymbol{p},B^z)>1,$$

which is inconsistent with the primitive solution $[A,B,C]$.

If $z>x>y$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},A)$:

$$(\boldsymbol{p}^xA^x+\boldsymbol{p}^yB^y=\boldsymbol{p}^zC^z)\Rightarrow(\boldsymbol{p}^{x-y}A^x+B^y=\boldsymbol{p}^{z-y}C^z)\Rightarrow\boldsymbol{cpf}(\boldsymbol{p},B^z)>1,$$

which is inconsistent with the primitive solution $[A,B,C]$.

If $z=x>y$, then for some $\boldsymbol{p}=\boldsymbol{cpf}(\boldsymbol{p},A)$:

$$(\boldsymbol{p}^xA^x+\boldsymbol{p}^yB^y=\boldsymbol{p}^zC^z)\Rightarrow(\boldsymbol{p}^{x-y}A^x+B^y=\boldsymbol{p}^{z-y}C^z)\Rightarrow\boldsymbol{cpf}(\boldsymbol{p},B^z)>1,$$

which is inconsistent with the primitive solution $[A,B,C]$. This is the wonderful proof.

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