THE PROOF OF THE GOLDBACH'S CONJECTURE

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Abstract

The proof of the Goldbach's Conjecture.

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Algebra of Sets, Diophantine Equations, Goldbach Conjecture, Greatest Common Divisor, Prime Numbers.

I. INTRODUCTION

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [1]

II. THE GOLDBACH'S CONJECTURE

Let

$$\{(2a+b)b\colon a \in [0,1,2,\dots] \wedge b \in [3,5,7,\dots]\} = \{9,15,21,25,27,33,35,39,45,49,\dots\} \wedge$$

$$\{3,5,7,\dots\} - \{9,15,21,25,27,33,35,39,45,49,\dots\} = \{3,5,7,11,13,17,19,23,29,31,\dots\} = \mathbb{P}.$$

Conjecture 1 (Goldbach Conjecture). For all $Z \in \{6,8,10,\dots\}$ and for some $X, Y \in \mathbb{P}$:

Proof.

$$\{6,8,10,\dots\} = \{\mathbf{6},12,\mathbf{18},24,\mathbf{30},36,\mathbf{42},48,\mathbf{54},60,\mathbf{66},72,\mathbf{78},84,\mathbf{90},\dots\} \cup$$

$$\{8,\mathbf{14},20,\mathbf{26},32,\mathbf{38},44,\mathbf{50},56,\mathbf{62},68,\mathbf{74},80,\mathbf{86},92,\mathbf{98},104,\mathbf{110},\dots\} \cup$$

$$\{\mathbf{10},16,\mathbf{22},28,\mathbf{34},40,\mathbf{46},52,\mathbf{58},64,\mathbf{70},76,\mathbf{82},88,\mathbf{94},100,\mathbf{106},112,\dots\}.$$

Thus

$$[3] \cup [\mathbf{9},\mathbf{15},\mathbf{21},\mathbf{27},\mathbf{33},\mathbf{39},\mathbf{45},\mathbf{51},\mathbf{57},\mathbf{63},\dots] \cup$$

$$[7,13,19,\mathbf{25},31,37,43,\mathbf{49},\mathbf{55},61,67,73,79,\mathbf{85},\mathbf{91},\dots] \cup$$

$$[5,11,17,23,29,\mathbf{35},41,47,53,59,\mathbf{65},71,\mathbf{77},83,89,\mathbf{95},\dots] = [3,5,7,\dots].$$

Hence

$$\{6\} = \{Z\colon\ Z = X + Y \wedge X = Y = 3\} \vee \{8\} = \{Z\colon\ Z = X + Y \wedge X = 3 \wedge Y = 5\} \vee$$

$$\{14,20,26,\dots\} = \{Z\colon\ Z = X + Y \wedge X \le Y \wedge X,Y \in \mathbb{P} \wedge X,Y \in [7,13,19,\mathbf{25},31,\dots]\} \vee$$

$$\{10,16,22,\dots\} =$$

$$\{Z\colon\ Z = X + Y \wedge X \le Y \wedge X,Y \in \mathbb{P} \wedge X,Y \in [5,11,17,23,29,\mathbf{35},\dots]\} \vee$$

$$\{12,18,24,\dots\} =$$

$$\{Z\colon\ Z = X + Y \wedge X,Y \in \mathbb{P} \wedge X \in [5,11,17,23,29,\mathbf{35},\dots] \wedge Y \in [7,13,19,\mathbf{25},31,\dots]\},$$

whence it implies that for all $Z \in \{6,8,10,\dots\}$ and for some $X,Y \in \mathbb{P}$: $Z = X + Y$.

Therefore we obtain

$$\{18,24,30,36,42,48,54,60,66,\dots\} =$$

$$\{Z\colon\ Z = x + y \wedge x \le y \wedge x,y \in [\mathbf{9},\mathbf{15},\mathbf{21},\mathbf{27},\mathbf{33},\dots]\} \vee$$

$$\{34,40,46,52,58,64,70,\dots\} =$$

$$\{Z\colon\ Z = x + y \wedge x < y \wedge x \in [\mathbf{9},\mathbf{15},\mathbf{21},\dots] \wedge y \in [\mathbf{25},31,37,\dots] - \mathbb{P}\} \vee$$

$$\{44,50,56,62,68,74,80,\dots\} =$$

$$\{Z\colon\ Z = x + y \wedge x < y \wedge x \in [\mathbf{9},\mathbf{15},\mathbf{21},\dots] \wedge y \in [\mathbf{35},41,47,\dots] - \mathbb{P}\}.$$

Thus for all $Z \in \{18,20,22,\dots\} - \{20,22,26,28,32,38\}$ and for some $x,y \in \{3,5,7,\dots\} - \mathbb{P}$: $Z = x + y$.

Finally we get

$$\{18,24,30,36,42,48,54,60,66,\dots\} =$$

$$\{Z\colon\ Z = x + y \wedge x \le y \wedge x,y \in [\mathbf{9},\mathbf{15},\mathbf{21},\mathbf{27},\mathbf{33},\dots]\} \Longrightarrow$$

$$\{6,8,10,12,14,16,18,20,22,\dots\} =$$

$$\{Z\colon Z = X + Y \wedge X \le Y \wedge X,Y \in [3,5,7,\dots] - [9,15,21,25,27,33,35,39,45\dots]\}.$$

This is the proof.

References

[1] https://en.wikipedia.org/wiki/Goldbach%27s_conjecture