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The proof of The Jeśmanowicz's Conjecture.
Disproof The Oesterlé-Masser Conjecture (The ABC Conjecture).

Cover Letter

Kenneth Alan Ribet

Lublin, 20 December 2015

Dear Professor Dr. Ken Ribet,

I would like to request You kindly to publish my work titled:

SEVERAL TREASURES OF THE QUEEN OF MATHEMATICS,
from the file named: lwg26stqm.pdf, sent here.

With respect and appreciation,
Leszek Guła

SEVERAL TREASURES OF THE QUEEN OF MATHEMATICS

By Leszek W. Guła

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June 1997 and September 2014 – November 2015 and 11 December 2015

Abstract

A new method of solving the equation $g = z^2 - y^2$ with given g.

The proper proof of The Fermat's Last Theorem (FLT)

Two complete proofs of The Beal's Conjecture.

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Disproof The Oesterlé–Masser Conjecture (The ABC Conjecture).

MSC: Primary - 11A41, 11D41, 11D45; Secondary - 11D61, 11D75, 11D85.

Keywords

ABC Conjecture, Algebra of Sets, Diophantine Equations, Diophantine Inequalities, Exponential Equations, Fermat Equation, Greatest Common Divisor, Newton Binomial Formula, Trinomial Square.

Dedicatory

I Dedicate this to My Wife

I. INTRODUCTION

The Theorem 1 (the new method) is dated 03 and 04 June 1997.

The Fermat's Last Theorem (FLT) is a famous theorem. It is easy to see that if $A^n + B^n = C^n$ then either $A, B,$ and C are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor). You could then restate FLT by saying that $A^n + B^n = C^n$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [10]

Beal has formulated a conjecture in number theory on which he has been working for several years. The Beal Conjecture. Let $A, B, C, x, y,$ and z be positive integers with $x; y; z > 2$. If $A^x + B^y = C^z$ then $A, B,$ and C have a common factor. [2] Or,…- below.- slightly restated also, with [9].

The Erdős-Straus Conjecture concerns the Diophantine Equations. The type of Diophantine equation discussed in this paper concerns Egyptian fractions, which deal with the representation of rational numbers as the sum of three unit fractions. [3]

The Jeśmanowicz's Conjecture [1] concerns the Diophantine Equation.

The ABC Conjecture concerns the equation $a + b = c$.

II. THE NEW METHOD OF SOLVING THE EQUATION $g = z^2 - y^2$ WITH GIVEN g.

Theorem 1. *For each* $g \in \{8,12,16, \dots\}$ *or for each* $g \in \{3,5,7, \dots\}$ *there exist finitely many pairs* (z, y) *of positive integers such that*:

$$g = \left(\frac{g + d^2}{2d}\right)^2 - \left(\frac{g - d^2}{2d}\right)^2 = z^2 - y^2 = (z + y)(z - y) = \frac{g}{d}(z - y) = \frac{g}{d} d = g,$$

where $d|g$ *and* $d < \sqrt{g}$ *and* - $d, \frac{g}{d} \in \{2,4,6, \dots\}$ *with even* g *or* $d \in \{1,3,5, \dots\}$ *with odd* g.

Proof of The Main Theorem. For each $g \in \{8,12,16, \dots\}$ or for each $g \in \{3,5,7, \dots\}$ there exist finitely many of divisors d of fixed g such that $d < \sqrt{g}$ and $d, \frac{g}{d} \in \{2,4,6, \dots\}$ with even g, or $d \in \{1,3,5, \dots\}$ with odd g:

$$\left(\frac{g + d^2}{2d} = z \ \wedge \ \frac{g - d^2}{2d} = y = z - d\right).$$

The number of the pairs (z, y) is finite because $d|g$ with fixed g. This is the proof.

Theorem 2. *Let* u *and* v *be two relatively prime natural numbers such that* $u - v$ *is positive and odd. Then* $(u^2 - v^2, 2uv, u^2 + v^2)$ *is a primitive Pythagorean triple, and each primitive Pythagorean triple arises in this way for some* u, v [6], *that is to say for all primitive Pythagorean triple there exists different and only one shared pair* (u, v).

III. THE PROPER PROOF OF THE FERMAT'S LAST THEOREM

Theorem 3 (Femat Last Theorem). *For all* $n \in \{3,4,5, \dots\}$ *and for all* $A, B, C \in \{1,2,3, \dots\}$:

$$A^n + B^n \neq C^n.$$

Proof. Suppose that for some $n \in \{3,4,5, \dots\}$ *and for some* $A, B, C \in \{1,2,3, \dots\}$:

$$A^n + B^n = C^n.$$

Then

$$(A + B > C \ \wedge \ A^2 + B^2 > C^2 \ \wedge \ \dots \ \wedge \ A^{n-1} + B^{n-1} > C^{n-1}),$$

otherwise $A^n + B^n < C^n$.

Every even number which is not the power of the number 2 has odd prime divisor, hence sufficient that we prove FLT for $n = 4$ and for odd prime numbers $n \in \mathbb{P}$. Fermat did not proved his own theorem for $n = 4$. [4].

We assume that the numbers $A, B,$ and C are co-prime. Then only one number out of (A, B, C) is even, which is obviously.

Without loss for this proof we assume that $A, C - B \in \{1,3,5, \dots\}$ and that $4 \nmid B, C$.

A. Proof For $n = 4$.

For some $B \in \{6,10,14, \dots\}$ and for some $C, A, \nu \in \{1,3,5, \dots\}$, where B, C, A are co-prime:

$$\{[B^4 = (C - A + 2\nu)^4 = (C - A + A)^4 - A^4 \ \wedge \ A^4 = (C - B + B)^4 - B^4] \Longrightarrow$$

$$[(C - A)^2 2\nu + \frac{3}{2}(C - A)(2\nu)^2 + (2\nu)^3 + \frac{4\nu^4}{C - A} = (C - A)^2 A + \frac{3}{2}(C - A)A^2 + A^3 \ \wedge$$

$$(C - B)^2 2\nu + \frac{3}{2}(C - B)(2\nu)^2 + (2\nu)^3 + \frac{4\nu^4}{C - B} = (C - B)^2 B + \frac{3}{2}(C - B)B^2 + B^3]\}.$$

It is known that

$$A^2 + B^2 > C^2 \Longrightarrow (2\nu)^2 > 2(C - A)(C - B).$$

From the above it follows that for some co-prime $c, d, e \in \{1,3,5, \dots\}$: $cde = \nu$, inasmuch as

$$(4d^4 = C - A \ \wedge \ c^4 = C - B).$$

Hence

$$(c^4 + 2cde = A \ \wedge \ 4d^4 + 2cde = B).$$

Therefore for some co-prime $c, d, e \in \{1,3,5, \dots\}$:

$$(2cde + 4d^4)^4 = [(4d^4 + A)^2]^2 - (A^2)^2 = [(4d^4 + A)^2 + A^2](2d^4 + A)8d^4 \Longrightarrow$$

$$2(ce + 2d^3)^4 = [(4d^4 + A)^2]^2 - (A^2)^2 = [(4d^4 + A)^2 + A^2](2d^4 + A).$$

We assume that for some co-prime $z, w, x \in \{1,3,5, \dots\}$ and for some $y \in \{6,10,14, \dots\}$:

$$[zw = ce + 2d^3 \ \wedge \ x + y = 2d^4 + A + 2d^4 \ \wedge \ x = 2d^4 + A \ \wedge \ y = 2d^4 \ \wedge$$

$$4 \nmid y \ \wedge \ 2(zw)^4 = ((x + y)^2 + (x - y)^2)x = 2(x^2 + y^2)x \ \wedge$$

$$z^4 w^4 = (x^2 + y^2)x \ \wedge \ (z^2)^2 = (x^2 + y^2) \ \wedge \ w^4 = x] \Longrightarrow 4|y,$$

which is inconsistent with $4 \nmid y$. This is the proper proof for $n = 4$.

Remark 1. This assumption $(A^2)^2 + (B^2)^2 = (C^2)^2$ is false because $B \notin \{6,10,14, \dots\}$ in view of $2uv(u^2 + v^2)\sqrt{2} = B$. On the strength of the Theorem 2: $(u^2 + v^2)^2 - (2uv)^2 = (u^2 - v^2)^2 = A^2$ and $(u^2 + v^2)^2 + (2uv)^2 = C^2 \equiv 0$, which is obviously. Moreover on the strength of the Theorem 1 $[u^2 + v^2 = (uv)^2 - 1 \ \vee \ u^2 + v^2 = u^2 - v^2] \equiv 0$. This is the remark 1.

B. Proof For $n \in \mathbb{P}$. General Deductions. We assumue that $n \mid A$ in view of $[5]$.

For some $B \in \{6,10,14,\dots\}$ and for some $C, A, \nu \in \{1,3,5,\dots\}$, where B, C, A are co-prime:

$$B-(C-A)=2\nu=A-(C-B) \wedge A+B-2\nu=C.$$

For some $n \in \mathbb{P}$ and some co-prime $A, C, B \in \{1,2,3,\dots\}$ such that $A, C-B \in \{1,3,5,\dots\}$:

$$[A^n=(C-B+2\nu)^n=(C-B+B)^n-B^n \wedge$$

$$B^n=(C-A+2\nu)^n=(C-A+A)^n-A^n \wedge$$

$$(A+B-B)^n+B^n=(A+B-2\nu)^n=C^n].$$

Thus for some co-prime $C, B, A \in \{1,2,3,\dots\}$ such that $C-B, A \in \{1,3,5,\dots\}$:

$$\{(C-B)^{n-2}\nu+(n-1)(C-B)^{n-3}\,\nu^2+\cdots+2^{n-2}\nu^{n-1}+\frac{2^{n-1}\nu^n}{n(C-B)}=$$

$$\frac{B}{2}\left[(C-B)^{n-2}+\frac{n-1}{2}(C-B)^{n-3}B+\cdots+B^{n-2}\right] \wedge$$

$$(C-A)^{n-2}2\nu+\frac{n-1}{2}(C-A)^{n-3}\,(2\nu)^2+\cdots+(2\nu)^{n-1}+\frac{(2\nu)^n}{n(C-A)}=$$

$$A\left[(C-A)^{n-2}+\frac{n-1}{2}(C-A)^{n-3}A+\cdots+A^{n-2}\right] \wedge$$

$$(A+B)^{n-2}(-B)+\frac{n-1}{2}(A+B)^{n-3}\,(-B)^2+\cdots+(-B)^{n-1}=$$

$$(A+B)^{n-2}(-2\nu)+\frac{n-1}{2}(A+B)^{n-3}\,(-2\nu)^2+\cdots+(-2\nu)^{n-1}+\frac{(-2\nu)^n}{n(A+B)}\}. \quad [5]$$

Thus for some $n \in \mathbb{P}$ and for some co-prime $e, m, c, h \in \{1,3,5,\dots\}$:

$$[nemch=\nu \wedge n \nmid emch \wedge h^n=C-A \wedge n^{n-1}c^n=C-B].$$

B.1. Proof For Odd $A, B, C-B$.

For some $n \in \mathbb{P}$ and some co-prime $e, m\, c, h \in \{1,3,5,\dots\}$ with $n \nmid emch$:

$$[n^{n-1}c^n+2nemch=A \wedge h^n+2nemch=B \wedge$$

$$2^n m^n=A+B=n^{n-1}c^n+h^n+4nemch \wedge n^{n-1}c^n+B=C] \Longrightarrow$$

$$[2^n m^n-h^n=n^{n-1}c^n+4nemch \wedge n\,|2m-h \wedge n^2 \mid 2^n m^n-h^n] \Longrightarrow n \mid emch,$$

which is inconsistent with $n \nmid emch$. ♠

B.2. Proof For Even $B, C-A$.

For some $n \in \mathbb{P}$ and some co-prime $e, m, c, h \in \{1,3,5, \dots\}$ with $n \nmid emch$:

$$[n^{n-1}c^n + 2nemch = A \ \wedge \ 2^n h^n + 2nemch = B \ \wedge$$

$$m^n = A + B = \ n^{n-1}c^n + 2^n h^n + 4nemch \ \wedge \ n^{n-1}c^n + B = C] \Longrightarrow$$

$$[m^n - 2^n h^n = \ n^{n-1}c^n + 4nemch \ \wedge \ n \mid m - 2h \ \wedge \ n^2 \mid \ m^n - 2^n h^n] \Rightarrow n \mid emch,$$

which is inconsistent with $n \nmid emch$. ♠ This is the proper proof for $n \in \mathbb{P}$. This is the proof.

Remark 2. If $C, A \in \{3^2, 5^2, 7^2, \dots\}$, then for some $n \in \mathbb{P}$ and for some $p, q, w, r, x \in \{1,3,5, \dots\}$ such that $n \mid pq$ and $p > q$ and $w > r$ and p, q, w, r, x are co-prime (mutually relatively prime):

$$\{[(2pq)^n = B^n = \left(C^{\frac{n}{2}}\right)^2 - \left(A^{\frac{n}{2}}\right)^2 \ \wedge \ C = (wr)^2 \ \wedge \ A = x^2 \wedge$$

$$\frac{(2pq)^n + (x^2)^n}{2pq + x^2} = \frac{(2pq)^n + (x^2)^n}{(r^2)^n} = \frac{(w^2 r^2)^n}{(r^2)^n} = (w^2)^n \ \wedge$$

$$(r^n)^2 - x^2 = 2pq \ \wedge \ (2 \mid pq \equiv 0)] \in \mathbf{0}\}.$$

The proof is incomplete because it does not include the case for $C \in \{6,10,14, \dots\}$ and it does not include the cases: $C, A \in \{3,5,7, \dots\} \setminus \{3^2, 5^2, 7^2, \dots\}$ and $(n \mid A \ \vee \ n \mid C)$. This is the remark 2.

IV. The Proof Of The Beal's Conjecture

Conjecture 1 (Beal Conjecture). For some $x, y, z \in \{3,4,5 \dots\}$ and for some $A, B, C \in \{1,2,3, \dots\}$ such that $A, B,$ and C have the common factor $\boldsymbol{p}k \geq 2$:

$$A^x + B^y = C^z.$$

Or – For all $x, y, z \in \{3,4,5 \dots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions $[A, B, C]$ in $\{1,2,3, \dots\}$.

Proof . Lemma 1. For all $n \in \{3,4,5, \dots\}$ and for all $a, b \in \{1,2,3, \dots\}$ with $a > b$ and $\boldsymbol{gcd}(a, b) = 1$, and for some $c \in \{1,2,3, \dots\}$ and for consecutive $k \in \{1,2,3, \dots\}$:

$$\left[\frac{a^n - b^n}{a - b} = c \ \wedge \ \boldsymbol{gcd}(a, c) = \boldsymbol{gcd}(b, c) = 1 \wedge \left((a - b)c\right)^k = \boldsymbol{p}k\right] \Longrightarrow$$

$$\left[a\left((a - b)c\right)^k\right]^n - \left[b\left((a - b)c\right)^k\right]^n = \left((a - b)c\right)^{nk+1} = B^{nk+1} = C^n - A^n.$$

This is the lemma 1.

Suppose that for some $n \in \{3,4,5, \dots\}$ and for some $k \in \{1,2,3, \dots\}$ the equation

$$A^n + B^{nk+1} = C^n$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \ldots\}$.

Then for some prime number $\boldsymbol{p} \geq 2$ we get:

$$\{\boldsymbol{p}^n A^n + \boldsymbol{p}^{kn+1} B^{kn+1} = \boldsymbol{p}^n C^n \land [\boldsymbol{gcd}(\boldsymbol{p}, A^n) > 1 \lor \boldsymbol{gcd}(\boldsymbol{p}, C^n) > 1]\}.$$

Hence – For some prime number $\boldsymbol{p} \geq 2$:

$$\{A^n + \boldsymbol{p}^{kn-n+1} B^{kn+1} = C^n \land [\boldsymbol{gcd}(\boldsymbol{p}, A^n) > 1 \lor \boldsymbol{gcd}(\boldsymbol{p}, C^n) > 1]\}.$$

Thus $\boldsymbol{gcd}(A, B, C) > 1$, which is inconsistent with $A, B,$ and C are co-prime.

Suppose that for some $n \in \{3,4,5, \ldots\}$ and for some $k \in \{1, 2,3, \ldots\}$ the equation

$$A^{nk} + B^{nk+1} = C^n$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \ldots\}$.

Then for some prime number $\boldsymbol{p} \geq 2$ we get:

$$\{\boldsymbol{p}^{nk} A^{nk} + \boldsymbol{p}^{kn+1} B^{kn+1} = \boldsymbol{p}^n C^n \land [\boldsymbol{gcd}(\boldsymbol{p}, A^{nk}) > 1 \lor \boldsymbol{gcd}(\boldsymbol{p}, B^{nk+1}) > 1]\}.$$

Hence – For some prime number $\boldsymbol{p} \geq 2$:

$$\{\boldsymbol{p}^{nk-n} A^{nk} + \boldsymbol{p}^{kn-n+1} B^{kn+1} = C^n \land [\boldsymbol{gcd}(\boldsymbol{p}, A^{nk}) > 1 \lor \boldsymbol{gcd}(\boldsymbol{p}, B^{nk+1}) > 1]\}.$$

Thus $\boldsymbol{gcd}(A, B, C) > 1$, which is inconsistent with $A, B,$ and C are co-prime.

Example 1. For all $n \in \{3,4,5, \ldots\}$ and all $a \in \{2,3,4, \ldots\}$ and for $b = 1$ and for $k = 2$ and for some $c \in \{1, 2,3, \ldots\}$ we will have

$$[(a^n - 1)^{2n} + (a^n - 1)^{2n+1} = (a(a^n - 1)^2)^n \ [8]\,] \land (a^n - 1)^2 = \left((a - 1)c\right)^k = \boldsymbol{p}k.$$

This is the example 1.

Lemma 2. For some $z, x \in \{3,4,5, \ldots\}$ and for some $a, b, c \in \{2,3,4, \ldots\}$ and for consecutive $k \in \{1, 2,3, \ldots\}$ with $\boldsymbol{gcd}(z, x) = \boldsymbol{gcd}(a, b) = 1$:

$$[a^z - b^x = c \land \boldsymbol{gcd}(a, c) = \boldsymbol{gcd}(b, c) = 1 \land c^{zxk} = \boldsymbol{p}k] \Longrightarrow$$

$$(ac^{xk})^z - (bc^{zk})^x = c^{zxk+1} = B^{zxk+1} = C^z - A^x.$$

This is the lemma 2.

Suppose that for some $x, z \in \{3,4,5, \ldots\}$ with $z > x,$ and for some $k \in \{1, 2,3, \ldots\}$ the equation

$$A^x + B^{xz+1} = C^z$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \ldots\}$.

Then for some prime number $\boldsymbol{p} \geq 2$ we get:

$$\{\boldsymbol{p}^x A^x + \boldsymbol{p}^{xz+1} B^{xz+1} = \boldsymbol{p}^z C^z \ \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, B^{xz+1}) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, C^z) > 1]\}.$$

Hence – For some prime number $\boldsymbol{p} \geq 2$:

$$\{A^x + \boldsymbol{p}^{xz-x+1} B^{xz+1} = \boldsymbol{p}^{z-x} C^z \ \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, B^{xz+1}) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, C^z) > 1]\}.$$

Thus $\boldsymbol{gcd}(A, B, C) > 1$, which is inconsistent with $A, B,$ and C are co-prime.

Lemma 3. For all $n \in \{3,4,5, \ldots\}$ and for all $a, b \in \{1,2,3, \ldots\}$ with $a > b$ and $\boldsymbol{gcd}(a, b) = 1$, and for some $C \in \{1, 2,3, \ldots\}$ and for consecutive $k \in \{1, 2,3, \ldots\}$:

$$[a^n + b^n = C \ \wedge \ \boldsymbol{gcd}(a, C) = \boldsymbol{gcd}(b, C) = 1 \wedge C^k = \boldsymbol{p}k] \Longrightarrow$$

$$(aC^k)^n + (bC^k)^n = C^{nk+1} = A^n + B^n.$$

This is the lemma 3.

Suppose that for some $n \in \{3,4,5, \ldots\}$ and for some $k \in \{1, 2,3, \ldots\}$ the equation

$$A^n + B^n = C^{nk+1}$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \ldots\}$.

Then for some prime number $\boldsymbol{p} \geq 2$ we get:

$$\{\boldsymbol{p}^n A^n + \boldsymbol{p}^n B^n = \boldsymbol{p}^{nk+1} C^{nk+1} \ \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, A^n) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, B^n) > 1]\}.$$

Hence – For some prime number $\boldsymbol{p} \geq 2$:

$$\{A^n + B^n = \boldsymbol{p}^{kn-n+1} C^{nk+1} \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, A^n) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, B^n) > 1]\}.$$

Thus $\boldsymbol{gcd}(A, B, C) > 1$, which is inconsistent with $A, B,$ and C are co-prime.

Suppose that for some $n \in \{3,4,5, \ldots\}$ and for some $k \in \{1, 2,3, \ldots\}$ the equation

$$A^{nk} + B^n = C^{nk+1}$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \ldots\}$.

Then for some prime number $\boldsymbol{p} \geq 2$ we get:

$$\{\boldsymbol{p}^{nk} A^{nk} + \boldsymbol{p}^n B^n = \boldsymbol{p}^{nk+1} C^{nk+1} \ \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, A^{nk}) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, C^{nk+1}) > 1]\}.$$

Hence – For some prime number $\boldsymbol{p} \geq 2$:

$$\{\boldsymbol{p}^{nk-n} A^{nk} + B^n = \boldsymbol{p}^{kn-n+1} C^{nk+1} \ \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, A^{nk}) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, C^{nk+1}) > 1]\}.$$

Thus $\boldsymbol{gcd}(A, B, C) > 1$, which is inconsistent with A, B, and C are co-prime.

Lemma 4. For all $x, y \in \{3,4,5,...\}$ and for all $a, b \in \{1,2,3,...\}$ with $x > y$ and $a > b$ and $\boldsymbol{gcd}(x, y) = \boldsymbol{gcd}(a, b) = 1$ and for consecutive $k \in \{1,2,3,...\}$:

$$[a^x + b^y = C \ \wedge \ \boldsymbol{gcd}(a, C) = \boldsymbol{gcd}(b, C) = 1 \wedge C^{xyk} = \boldsymbol{p}k] \Longrightarrow$$

$$(aC^{yk})^x + (bC^{xk})^y = C^{xyk+1} = A^x + B^y.$$

This is the lemma 4.

Remark 3. The lemmas gives all solutions of the equation $A^x + B^y = C^z$ such that A, B, and C have the common prime factor $\boldsymbol{p} \geq 2$ inasmuch as $\boldsymbol{p} \mid \boldsymbol{p}k$. The solutions $[A, B, C]$ is infinite. ♠

Suppose that for some $x, y \in \{3,4,5,...\}$ with $x > y$, and for some $k \in \{1,2,3,...\}$ the equation

$$A^x + B^y = C^{xyk+1}$$

has primitive solutions $[A, B, C]$ in $\{1,2,3,...\}$.

Then for some prime number $\boldsymbol{p} \geq 2$ we get:

$$\{\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^{xyk+1} C^{xyk+1} \ \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, A^x) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, C^{xyk+1}) > 1]\}.$$

Hence – For some prime number $\boldsymbol{p} \geq 2$:

$$\{\boldsymbol{p}^{x-y} A^x + B^y = \boldsymbol{p}^{xyk-y+1} C^{xyk+1} \ \wedge \ [\boldsymbol{gcd}(\boldsymbol{p}, A^x) > 1 \ \vee \ \boldsymbol{gcd}(\boldsymbol{p}, C^{xyk+1}) > 1]\}.$$

Thus $\boldsymbol{gcd}(A, B, C) > 1$, which is inconsistent with A, B, and C are co-prime.

This is the proof.

Corollary 1. *For some* $x, y, z \in \{3,4,5 ...\}$ *the equation*

$$A^x + B^y = C^z$$

has solutions $[A, B, C]$ *in* $\{1,2,3,...\}$ *such that* A, B, *and* C *have the common prime factor* $\boldsymbol{p} \geq 2$.

Corollary 2. *For all* $x, y, z \in \{3,4,5 ...\}$ *and for all* $A, B, C \in \{1,2,3,...\}$ *such that* A, B, *and* C *are co-prime*: $A^x + B^y \neq C^z$.

V. THE PROOF OF THE ERDŐS-STRAUS CONJECTURE

Conjecture 2 (Erdős–Straus Conjecture). For all $n \in \{2,3,4,...\}$ and for some $a, b, c \in \{1,2,3,...\}$:

$$\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Proof. For all $n \in \{2,4,6,...\}$ and for some $a, b, c \in \{1,2,3,...\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ \wedge\ \frac{n}{2}=a\ \wedge\ \frac{n+2}{2}=b\ \wedge\ \frac{n(n+2)}{4}=\mathrm{c}\right].\ ♠$$

For all $n \in \{3,7,11,\dots\}$ and for some $a,c,b \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{c}+\frac{1}{b}\wedge\frac{n+1}{2}=a=c\ \wedge\frac{n(n+1)}{4}=b\right]. \qquad (1)$$

Thus for all $n \in \{3,9,15,21,27,33,39,45,51,57,63,\dots\}$ and for some $a,c,b \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{c}+\frac{1}{b}\ \wedge\ \frac{2n}{3}=a=c\ \wedge\ n=b\right],$$

and for all $n \in \{7,21,35,49,63,77,91,105,119,133,147,\dots\}$ and for some $a,c,b \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{c}+\frac{1}{b}\ \wedge\ \frac{4n}{7}=a=c\ \wedge\ 2n=b\right],$$

and for all $n \in \{11,33,55,77,99,121,143,165,187,209,\dots\}$ and for some $a,c,b \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{c}+\frac{1}{b}\ \wedge\ \frac{6n}{11}=a=c\ \wedge\ 3n=b\right],\dots.$$

For all $n \in \{5,13,21,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\wedge\frac{3n+1}{4}=a\ \wedge\frac{n(3n+1)}{4}=b\ \wedge\frac{3n+1}{8}=c\right]. \qquad (2)$$

For all $n \in \{5,11,17,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\wedge\frac{n+1}{3}=a\ \wedge\ n=b\ \wedge\frac{n(n+1)}{3}=c\right]. \qquad (3)$$

On the strength of [3] for all $n \in \{97,111,125,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\wedge\frac{2(n+1)}{7}=a\ \wedge\ 2n=b\ \wedge\frac{2n(n+1)}{7}=c\right]. \qquad (4)$$

For all $n \in \{3,17,31,\dots\}$ and for some $c,x \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{2n}+\frac{1}{c}+\frac{1}{c+x}\wedge\frac{4n^2-1}{7}=x\ \wedge\ \frac{2n+1}{7}=c\right]. \qquad (5)$$

On the strength of [3] for all $n \in \{13,33,53,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\wedge\frac{3n+1}{10}=a\ \wedge\frac{n(3n+1)}{4}=b\ \wedge\frac{3n+1}{2}=c\right]. \qquad (6)$$

The key of this proof is the following sum of the subsets, which should be written in columns to determine the missing odd prime numbers, namely

$$\{5,9,13,\dots\} =$$

$$\{5,25,45,\dots\} \cup \{9,29,49,\dots\} \cup \{13,33,53,\dots\} \cup \{17,37,57,\dots\} \cup \{21,41,61,\dots\}.$$

From $(2),(3),(4),(5),\text{and } (6)$ we will have by analogy yes, as well from (1).

Therefore, and on the strength of the Theorem 1 and the Theorem 2 we get -

for all $n \in \{n\colon\ n = 337 + 120k \ \wedge\ k \in [0,1,2,\dots] \ \wedge\ only\ \ 1,n \mid n\}$

or for all $n \in \{n\colon\ n = 1009 + 120k \ \wedge\ k \in [0,1,2,\dots] \ \wedge\ only\ \ 1,n \mid n\}$

or for all $n \in \{n\colon\ n = 1201 + 120k \ \wedge\ k \in [0,1,2,\dots] \ \wedge\ only\ \ 1,n \mid n\}$

or for some $m,x,c \in \{1,2,3,\dots\}$ and for some $d \in \{2,4,6,\dots\}$ such that $2nm > d$:

$$\left[\frac{4}{n} = \frac{1}{nm} + \frac{1}{x+c} + \frac{1}{c} \implies (4m-1)c^2 + [(4m-1)x - 2nm]c - nmx = 0\right] \implies$$

$$\left[\Delta = [(4m-1)x]^2 + (2nm)^2 = (n^2+m^2)^2 \ \wedge\ x = \frac{n^2-m^2}{4m-1} \ \wedge\ c = \frac{nm+m^2}{4m-1}\right] \vee$$

$$\left[\Delta = [(4m-1)x]^2 + (2nm)^2 = \left(\frac{(2nm)^2 + d^2}{2d}\right)^2 \ \wedge\ x = \frac{\frac{(2nm)^2}{2d} - \frac{d}{2}}{4m-1} \ \wedge\ c = \frac{nm + \frac{d}{2}}{4m-1}\right].$$

This is the proof.

Example 3. For $n = 337$ and for $m = 18$ and for some $x,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4m-1}{nm} = \frac{x+2c}{(x+c)c} \ \wedge\ 71c^2 + (71x - 12132)c - 6066x = 0\right] \implies$$

$$\left[\Delta = (71x)^2 + 12132^2 = (n^2+m^2)^2 \ \wedge\ x = \frac{337^2 - 18^2}{71} \ \wedge\ c = \frac{6066+324}{71} = 90\right].$$

Hence for all $n \in \{337,1011,1685,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge\ \frac{1685n}{337} = (x+c) = a \ \wedge\ 18n = b \ \wedge\ \frac{90n}{337} = c\right].$$

This is the example 3.

Example 4. For $n = 1009$ and for $m = 3$ and for some $x,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4m-1}{nm} = \frac{x+2c}{(x+c)c} \ \wedge\ 3c^2 + (3x - 6054)c - 3027x = 0\right] \implies$$

$$\left[\Delta = (3x)^2 + 6054^2 = (n^2 + m^2)^2 \ \wedge \ x = \frac{1009^2 - 3^2}{11} \ \wedge \ c = \frac{3027 + 9}{11} = 276\right].$$

Hence for all $n \in \{1009,3027,5045, \ldots\}$ and for some $a, b, c \in \{1,2,3, \ldots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{92828n}{1009} = (x + c) = a \ \wedge \ 3n = b \ \wedge \ \frac{276n}{1009} = c\right].$$

This is the example 4.

Example 5. For $n = 1201$ and for $m = 8$ and for some $x, c \in \{1,2,3, \ldots\}$:

$$\left[\frac{31}{8n} = \frac{x + 2c}{(x + c)c} \ \wedge \ 31c^2 + (31x - 19216)c - 9608x = 0\right] \Longrightarrow$$

$$\left[\Delta = (31x)^2 + 19216^2 = (n^2 + m^2)^2 \ \wedge \ x = \frac{1201^2 - 8^2}{31} \ \wedge \ c = \frac{nm + m^2}{31}\right].$$

Hence for all $n \in \{1201,3603,6005, \ldots\}$ and for some $a, b, c \in \{1,2,3, \ldots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{46839n}{1201} = a \ \wedge \ 8n = b \ \wedge \ \frac{312n}{1201} = c\right].$$

This is the example 5.

VI. The Proof Of The Jeśmanowicz's Conjecture

Conjecture 3 (Jesmanowicz Conjecture). For all $p, q \in \{0,1,2, \ldots\}$ and for all $x, r, s \in \{1,2,3, \ldots\}$ such that $p + q > 0$ and $r - s \in \{1,3,5, \ldots\}$ and $\boldsymbol{gcd}(r, s) = 1$:

$$[(r^2 - s^2)^{p+x} + (2rs)^x \neq (r^2 + s^2)^{q+x} \ \wedge \ (r^2 - s^2)^x + (2rs)^{p+x} \neq (r^2 + s^2)^{q+x}].$$

Proof. Suppose that for some $p, q \in \{0,1,2, \ldots\}$ and for some $x, r, s \in \{1,2,3, \ldots\}$ such that $p + q > 0$ and $r - s \in \{1,3,5, \ldots\}$ and $\boldsymbol{gcd}(r, s) = 1$:

$$[(r^2 - s^2)^{p+x} + (2rs)^x = (r^2 + s^2)^{q+x} \vee (r^2 - s^2)^x + (2rs)^{p+x} = (r^2 + s^2)^{q+x}].$$

On the strength of the Theorem 2 and of the Theorem 4 it must be

$$(p + x = 2 \ \wedge \ p = 0 \ \wedge \ x = 2 \ \wedge \ q + x = 2 \ \wedge \ q = 0) \ \Longrightarrow \ p + q = 0,$$

which is inconsistent with $p + q > 0$. This is the proof.

VII. Disproof The ABC Conjecture

Conjecture 4 (ABC Conjecture). For all $\epsilon > 0$ there exist only finitely many triples (a, b, c) of positive coprime integers, with $a + b = c > d^{1+\epsilon}$, where d denotes the product of the distinct prime factors of the product abc. [7]

Disproof. *For all* $n \in \{1,3,5,\ldots\}$ *and for all relatively prime* $A, B \in \{1,2,3,\ldots\}$ *and for some* $g \in \{1,2,3,\ldots\}$ [5] *and for some* $\epsilon \in \{1,2,3,\ldots\}$:

$$A^n + B^n = (A+B)\frac{A^n + B^n}{A+B} = (A+B)g = a + b = \epsilon = c < d^{1+\epsilon},$$

where d *denotes the product of the distinct prime factors of the product* abc. This is the disproof.

REFERENCES

[1]. Bobiński, Z., Kamiński, B.: WIADOMOŚCI MATEMATYCZNE XXXV, SERIES II, ROCZNIKI POLSKIEGO TOWARZYSTWA MATEMATYCZNEGO 1999 – http://main3.amu.edu.pl/~wiadmat/145-151_zb_wm35.pdf

[2]. Mauldin R. Daniel : A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem – http://www.ams.org/notices/199711/beal.pdf

[3]. Monks, M. and Velingker, A.: On the Erdős-Straus Conjecture: Properties of Solutions to its Underlying Diophantine Equation – http://www.cs.cmu.edu/~avelingk/papers/erdos_straus.pdf

[4]. Narkiewicz, W. : WIADOMOŚCI MATEMATYCZNE XXX.1, Annuals PTM, Series II, Warszawa 1993.

[5]. Guła, L. W.: The Truly Marvellous Proof – http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf

[6]. Husemöler, D.: Elliptic Curves, Second Edition, Springer, p. 7 – http://www.math.rochester.edu/people/faculty/doug/otherpapers/Husemoller.pdf

[7]. http://en.wikipedia.org/wiki/Abc_conjecture

[8]. http://en.wikipedia.org/wiki/Beal's_conjecture

[9]. http://www.ams.org/profession/prizes-awards/ams-supported/beal-prize

[10]. http://www.bealconjecture.com