THE PROOF OF THE ERDŐS-STRAUS CONJECTURE

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Abstract

The proof of The Erdős-Straus Conjecture.

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Algebra of Sets, Diophantine Equations, Elementary Number Theory, Pythagoras Equation, Trinomial Square.

Dedicatory

I Dedicate this to My Wife

I. INTRODUCTION

One important topic in number theory is the study of Diophantine equations, equations in which only integer solutions are permitted. The type of Diophantine equation discussed in this paper concerns Egyptian fractions, which deal with the representation of rational numbers as the sum of three unit fractions. [1] The Erdős–Straus Conjecture concerns the Diophantine Equations.

II. THE PROOF OF THE ERDŐS-STRAUS CONJECTURE

Conjecture (Erdős– Straus Conjecture). For all $n \in \{2,3,4,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Proof. For all $n \in \{2,4,6,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge\ \frac{n}{2} = a \ \wedge\ \frac{n+2}{2} = b \ \wedge\ \frac{n(n+2)}{4} = \mathrm{c}\right]. \ \spadesuit$$

For all $n \in \{3,7,11,15,19,23,27,31,35,39,43,47,51,55,\dots\}$ and for some $a,c,b \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \ \wedge\ \frac{n+1}{2} = a = c \ \wedge\ \frac{n(n+1)}{4} = b\right]. \ \spadesuit$$

For all $n \in \{5,13,21,29,37,45,53,61,69,77,85,93,101,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ \wedge\ \frac{3n+1}{4}=a\ \wedge\ \frac{n(3n+1)}{4}=b\ \wedge \frac{3n+1}{8}=c\right].\ \spadesuit$$

We assume that for all $n\in\{9,17,25,33,41,49,57,65,73,\ldots\}$ and for some $a,b,c,x\in\{1,2,3,\ldots\}$:

$$\left[\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ \wedge\ 2n=b\wedge\ 7ac=2n(a+c)\ \wedge\ a-c=x\right].$$

Thus for all $n\in\{9,17,25,33,41,49,57,65,73,81,89,97,105,113,\ldots\}$ and for some $c,x\in\{1,2,3,\ldots\}$ and for some $d\in\{2,4,6,\ldots\}$ such that $\frac{(4n)^2}{d}$ is even:

$$[\,4nc+2nx=7c^2+7cx\ \Longrightarrow -2nx-(4n-7x)c+7c^2=0]\ \wedge$$

$$\left[(4n)^2+(7x)^2=\left(\frac{(4n)^2+d^2}{2d}\right)^2=\Delta\right]\ \wedge$$

$$\left[7x=\frac{(4n)^2-d^2}{2d}\ \wedge\ c=\frac{-(7x-4n)+\sqrt{\Delta}}{14}\right]\ \wedge\ \frac{4}{n}=\frac{1}{c+x}+\frac{1}{2n}+\frac{1}{c}\ .\ \spadesuit$$

Theorem. For all $n\in\{3,5,7,\ldots\}$ and for some $c,x\in\{1,2,3,\ldots\}$ and for some $d\in\{2,4,6,\ldots\}$ such that $\frac{(4n)^2}{d}$ is even:

$$[\,4nc+2nx=7c^2+7cx\ \Longrightarrow -2nx-(4n-7x)c+7c^2=0]\ \wedge$$

$$\left[(4n)^2+(7x)^2=\left(\frac{(4n)^2+d^2}{2d}\right)^2=\Delta\right]\ \wedge$$

$$\left[7x=\frac{(4n)^2-d^2}{2d}\ \wedge\ c=\frac{-(7x-4n)+\sqrt{\Delta}}{14}\right]\ \wedge\ \frac{4}{n}=\frac{1}{c+x}+\frac{1}{2n}+\frac{1}{c}\,.$$

This is the proof.

REFERENCES

[1] http://www.cs.cmu.edu/~avelingk/papers/erdos_straus.pdf