THE PROOF OF THE JEŚMANOWICZ'S CONJECTURE

Leszek W. Guła

Lublin-POLAND

lwgula@wp.pl

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Abstract

The proof of The Jeśmanowicz's Conjecture.

MSC: Primary - 11D45; Secondary - 11D61; 11D75.

Keywords

Diophantine Equations, Diophantine Inequalities, Exponential Equations, Greatest Common Divisor, Pythagoras Equation.

I. INTRODUCTION

Theorem 1. *Let* r *and* s *be two relatively prime natural numbers such that* $r-s$ *is positive and odd. Then* $(r^2-s^2,\ 2rs,\ r^2+s^2)$ *is a primitive Pythagorean triple, and each primitive Pythagorean triple arises in this way for some* r,s [2] *that is to say for all primitive Pythagorean triple there exists only one and different pair* (r,s).

II. THE PROOF OF THE JEŚMANOWICZ'S CONJECTURE

Jeśmanowicz Conjecture. *For all* $p,q\in\{0,1,2,\ldots\}$ *and for all* $x,r,s\in\{1,2,3,\ldots\}$ *such that* $p+q>0$ *and* $r-s\in\{1,3,5,\ldots\}$ *and* $\boldsymbol{gcd}(r,s)=1$:

$$[(r^2-s^2)^{p+x}+(2rs)^x\neq(r^2+s^2)^{q+x}\ \wedge$$

$$(2rs)^{p+x}+(r^2-s^2)^x\neq(r^2+s^2)^{q+x}].$$

This is The Jeśmanowicz Conjecture (slightly restated with [1]) because FLT is true. [3]

Proof. Suppose that for some $p,q\in\{0,1,2,\ldots\}$ and for some $x,r,s\in\{1,2,3,\ldots\}$ such that $p+q>0$ and $r-s\in\{1,3,5,\ldots\}$ and $\mathbf{gcd}(r,s)=1$:

$$[(r^2-s^2)^{p+x}+(2rs)^x=(r^2+s^2)^{q+x}\ \vee$$

$$(2rs)^{p+x}+(r^2-s^2)^x=(r^2+s^2)^{q+x}].$$

On the strength of the Theorem 1 and of The Fermat's Last Theorem [3] it must be

$$(p+x=2\ \wedge\ x=2\ \wedge\ q+x=2)\Longrightarrow p=q=0,$$

which is inconsistent with $p+q>0$. ♠

REFERENCES

[1] Bobiński, Z., Kamiński, B.: WIADOMOŚCI MATEMATYCZNE XXXV, SERIES II, ROCZNIKI POLSKIEGO TOWARZYSTWA MATEMATYCZNEGO 1999 - http://main3.amu.edu.pl/~wiadmat/145-151_zb_wm35.pdf

[2] Husemöler, D. Elliptic Curves, Second Edition, Springer, p. 7 - http://www.math.rochester.edu/people/faculty/doug/otherpapers/Husemoller.pdf

[3] Guła, L. W.: The Proof of The Beal's Conjecture - http://www.ijmsea.com/admin/docs/1423144652ISSUE-4.pdf