THE PROOF OF THE JEŚMANOWICZ'S CONJECTURE

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Abstract

The proof of The Jeśmanowicz's Conjecture.

MSC: Primary - 11D45; Secondary - 11D61; 11D75.

Keywords

Diophantine Equations, Diophantine Inequalities, Exponential Equations, Greatest Common Divisor, Pythagoras Equation.

I. INTRODUCTION

Theorem 1. *Let* r *and* s *be two relatively prime natural numbers such that* $r - s$ *is positive and odd. Then* $(r^2 - s^2,\ 2rs,\ r^2 + s^2)$ *is a primitive Pythagorean triple, and each primitive Pythagorean triple arises in this way for some* r, s [2] *that is to say for all primitive Pythagorean triple there exists only one and different pair* (r, s).

II. THE PROOF OF THE JEŚMANOWICZ'S CONJECTURE

Jeśmanowicz Conjecture. *For all* $p, q \in \{0,1,2,\dots\}$ *and for all* $x, r, s \in \{1,2,3,\dots\}$ *such that* $p + q > 0$ *and* $r - s \in \{1,3,5,\dots\}$ *and* $\boldsymbol{gcd}(r, s) = 1$:

$$[(r^2 - s^2)^{p+x} + (2rs)^x \neq (r^2 + s^2)^{q+x} \ \wedge$$

$$(2rs)^{p+x} + (r^2 - s^2)^x \neq (r^2 + s^2)^{q+x}].$$

This is The Jeśmianowicz Conjecture (slightly restated with [1]) because FLT is true. [3]

Proof. Suppose that for some $p, q \in \{0,1,2,\dots\}$ and for some $x, r, s \in \{1,2,3,\dots\}$ such that $p + q > 0$ and $r - s \in \{1,3,5,\dots\}$ and $\mathbf{gcd}(r, s) = 1$:

$$[(r^2 - s^2)^{p+x} + (2rs)^x = (r^2 + s^2)^{q+x} \ \vee$$

$$(2rs)^{p+x} + (r^2 - s^2)^x = (r^2 + s^2)^{q+x}].$$

On the strength of the Theorem 1 and of The Fermat's Last Theorem [3] it must be

$$(p + x = 2 \ \wedge \ x = 2 \ \wedge \ q + x = 2) \Longrightarrow p = q = 0,$$

which is inconsistent with $p + q > 0$. ♠

REFERENCES

[1] Bobiński, Z., Kamiński, B.: WIADOMOŚCI MATEMATYCZNE XXXV, SERIES II, ROCZNIKI POLSKIEGO TOWARZYSTWA MATEMATYCZNEGO 1999 - http://main3.amu.edu.pl/~wiadmat/145-151_zb_wm35.pdf

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[3] Guła, L. W.: The Proof of The Beal's Conjecture - http://www.ijmsea.com/admin/docs/1423144652ISSUE-4.pdf