The Proof of The Fermat-Beal Theorem

Leszek W. Guła

Lublin-POLAND

lwgula@wp.pl

01 - 02 October 2014 – 30 November 2014 – 05 -08 December 2014

Abstract

The proof of The Beal's Conjecture.

Keywords

Beal Conjecture, Diophantine Equations, Fermat Theorem, Greatest Common Divisor, Number theory.

Dedicatory

Dedicated to my Family

I. INTRODUCTION

The famous Fermat's Last Theorem (FLT) assertion that for all $x \in \{3,4,5,\dots\}$ and for all $X, Y, Z \in \{1,2,3,\dots\}$: $X^x + Y^x \neq Z^x \in 1$. [2]

It is easy to see that if $X^x + Y^x = Z^x$ then either $X, Y, and\ Z$ are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor).

You could then restate FLT by saying that $X^x + Y^x = Z^x$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [1]

II. THE FERMAT-BEAL THEOREM

Theorem (Fermat – Beal Theorem). For all $x, y, z \in \{3,4,5,\dots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions in $\{1,2,3,\dots\}$.

Proof. Suppose that for some $x, y, z \in \{3,4,5,\dots\}$ the equation

$$A^x + B^y = C^z$$

has primitive solutions in $\{1,2,3,\dots\}$.

Thus for some $x, y, z \in \{3,4,5,\dots\}$ and for some coprime $A, B, C \in \{1,2,3,\dots\}$ and for some coprime $a, b, c \in \{1,2,3,\dots\}$ such that only one number out of (A, B, C) is even:

$$\{\left[A^x + \left(B^{\frac{y}{x}}\right)^x = \left(C^{\frac{z}{x}}\right)^x \wedge\ x \mid y,z\ \wedge\ y \neq z\right] \vee$$

$$\left[\left(A^{\frac{x}{y}}\right)^y +\ B^y = \left(C^{\frac{z}{y}}\right)^y \wedge\ y \mid x,z\ \wedge\ x \neq z\right] \vee$$

$$\left[\left(A^{\frac{x}{z}}\right)^z + \left(B^{\frac{y}{z}}\right)^z =\ C^z \wedge\ z \mid x,y\ \wedge\ x \neq y\right] \vee$$

$$[A^x +\ B^x = C^x \wedge\ x = y = z]\ \vee$$

$$[A^x + \left(B^{\frac{y}{x}}\right)^x = \left(C^{\frac{z}{x}}\right)^x \wedge\ x \nmid y,z\ \wedge\ b^x = B\ \wedge\ c^x = C\ \wedge$$

$$A^x + (b^y)^x = (c^z)^x\ \wedge\ \mathbf{gcd}(A,b) = \mathbf{gcd}(A,c) = 1]\ \vee$$

$$[\left(A^{\frac{x}{y}}\right)^y + B^y\ = \left(C^{\frac{z}{y}}\right)^y \wedge\ y \nmid x,z\ \wedge\ a^y = A\ \wedge\ c^y = C\ \wedge$$

$$(a^x)^y + B^y\ = (c^z)^y \wedge\ \mathbf{gcd}(a,B) = \mathbf{gcd}(B,c) = 1]\ \vee$$

$$[\left(A^{\frac{x}{z}}\right)^z + \left(B^{\frac{y}{z}}\right)^z\ =\ C^z\ \wedge\ z \nmid x,y\ \wedge\ a^z = A\ \wedge\ b^z = B\ \wedge$$

$$(a^x)^z + (b^y)^z\ = C^z \wedge\ \mathbf{gcd}(a,C) = \mathbf{gcd}(b,C) = 1]\ \vee$$

$$[A^x + \left(B^{\frac{y}{x}}\right)^x = \left(C^{\frac{z}{x}}\right)^x \wedge\ x \mid y\ \wedge\ x \nmid z\ \wedge\ y \neq z\ \wedge\ c^x = C\ \wedge$$

$$A^x + \left(B^{\frac{y}{x}}\right)^x\ = (c^z)^x\ \wedge\ \mathbf{gcd}(A,c) = \mathbf{gcd}(B,c) = 1]\ \vee$$

$$[A^x + \left(B^{\frac{y}{x}}\right)^x = \left(C^{\frac{z}{x}}\right)^x \wedge\ x \mid z\ \wedge\ x \nmid y\ \wedge\ y \neq z\ \wedge\ b^x = B\ \wedge$$

$$A^x + (b^y)^x = \left(C^{\frac{z}{x}}\right)^x \wedge\ \mathbf{gcd}(A,b) = \mathbf{gcd}(b,C) = 1]\ \vee$$

$$[\left(A^{\frac{x}{y}}\right)^y + B^y = \left(C^{\frac{z}{y}}\right)^y \wedge\ y \mid x\ \wedge\ y \nmid z\ \wedge\ x \neq z\ \wedge\ c^y = C\ \wedge$$

$$\left(A^{\frac{x}{y}}\right)^y + B^y =\ (c^z)^y\ \wedge\ \mathbf{gcd}(A,c) = \mathbf{gcd}(B,c) = 1]\ \vee$$

$$[\left(A^{\frac{x}{y}}\right)^y + B^y = \left(C^{\frac{z}{y}}\right)^y \wedge\ y \mid z\ \wedge\ y \nmid x\ \wedge\ x \neq z\ \wedge\ a^y = A\ \wedge$$

$$(a^x)^y + B^y = \left(C^{\frac{z}{y}}\right)^y \wedge\ \mathbf{gcd}(a,B) = \mathbf{gcd}(a,C) = 1]\ \vee$$

$$[\left(A^{\frac{x}{z}}\right)^z + \left(B^{\frac{y}{z}}\right)^z = C^z \wedge z \mid x \wedge z \nmid y \wedge x \neq y \wedge b^z = B \wedge$$

$$\left(A^{\frac{x}{z}}\right)^z + (b^y)^z = C^z \wedge \mathbf{gcd}(A, b) = \mathbf{gcd}(b, C) = 1] \vee$$

$$[\left(A^{\frac{x}{z}}\right)^z + \left(B^{\frac{y}{z}}\right)^z = C^z \wedge z \mid y \wedge z \nmid x \wedge x \neq y \wedge a^z = A \wedge$$

$$(a^x)^z + \left(B^{\frac{y}{z}}\right)^z = C^z \wedge \mathbf{gcd}(a, B) = \mathbf{gcd}(a, C) = 1]\},$$

which is false because FLT is true. ♠

The Last Case. For some $x, y, z \in \{3,4,5, \dots\}$ and for some coprime $X, Y, Z \in \{1,2,3, \dots\}$ and for some coprime $a, b, c \in \{1,2,3, \dots\}$ such that only one number out of (a, b, c) is even:

$$[(X = a^x \wedge Y = b^y \wedge Z = c^z \wedge a^{xx} + b^{xy} = c^{xz} \equiv 0) \in 0].$$

Thus for some $x, y, z \in \{3,4,5, \dots\}$ and for some coprime $a, b, c \in \{1,2,3, \dots\}$ and for some coprime $A, B, C \in \{1,2,3, \dots\}$ such that only one number out of (A, B, C) is even:

$$[(a^{xx} + b^{xy} = c^{xz} = A^x + B^y = C^z \equiv 0 \wedge a^x = A \wedge b^x = B \wedge c^x = C) \in 0],$$

inasmuch as the Fermat Equation is false. This is the proof.

References

[1]. http://www.bealconjecture.com

[2]. Guła, L. W.: http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf