

The Proof of The Beal's Conjecture

Leszek W. Gula

Lublin-POLAND

lwgula@wp.pl

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Abstract

1. The truly marvellous proof of The Fermat's Last Theorem (FLT).
2. Two false proofs of FLT for $n = 4$.
3. The incomplete proof of FLT for odd prime numbers $n \in \mathbb{P}$.
4. The proof of The Beal's Conjecture.
5. The Beal's Theorem.

Keywords

Common Prime Factor, Diophantine Equations, Fermat Equation, Greatest Common Divisor, Newton Binomial Formula.

Dedicatory

Dedicated to my Parents and my Brother

I. INTRODUCTION

The cover of this issue of the Bulletin is the frontispiece to a volume of Samuel de Fermat's 1670 edition of Bachet's Latin translation of Diophantus's *Arithmetica*. This edition includes the marginalia of the editor's father, Pierre de Fermat. Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation $x^2 + y^2 = z^2$ the marginal comment that hints at the existence of a proof (a *demonstratio sane mirabilis*) of what has come to be known as Fermat's Last Theorem. Diophantus's work had fired the imagination of the Italian Renaissance mathematician Rafael Bombelli, as it inspired Fermat a century later. [6]

Problem II.8 of the Diophantus's *Arithmetica* asks how a given square number is split into two other squares. Diophantus's shows how to solve this sum-of-squares problem for $k = 4$ and $u = 2$ [8], inasmuch as for all $k, u \in \{\dots - 2, -1, 0, 1, 2, \dots\}$:

$$k^2 = \left(\frac{2ku}{u^2 + 1}\right)^2 + \left[\frac{k(u^2 - 1)}{u^2 + 1}\right]^2. \quad [4]$$

Thus for all relatively prime natural numbers u, v such that $u - v \in \{1, 3, 5, \dots\}$:

$$(u^2 + v^2)^2 = u^4 - 2u^2v^2 + v^4 + 4u^2v^2 = (u^2 - v^2)^2 + (2uv)^2.$$

We have a primitive Pythagorean triple $(u^2 - v^2, 2uv, u^2 + v^2) = (x, y, z)$ the primitive triple because the numbers x, y , and z are coprime.

Around 1637, Fermat wrote his Last Theorem in the margin of his copy of the Arithmetica next to Diophantus sum-of-squares problem: it is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain. In number theory, Fermat's Last Theorem (FLT) states that no three positive integers $A, B, \text{ and } C$ can satisfy the equation $A^n + B^n = C^n$ for any integer value of n greater than two. [8]

It is easy to see that if $A^n + B^n = C^n$ then either $A, B, \text{ and } C$ are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor).

You could then restate FLT by saying that $A^n + B^n = C^n$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [1]

It is known that for some coprime $x, y, z \in \{3, 4, 5, \dots\}$:

$$[x^2 + y^2 = z^2 \wedge (x + y)^2 + (x - y)^2 = 2z^2], \quad (1)$$

where z is odd because for all $a, b \in \{0, 1, 2, 3, \dots\}$: the number $\frac{(2a+1)^2 + (2b+1)^2}{2}$ is odd.

II. THE FERMAT'S LAST THEOREM AND THE BEAL'S CONJECTURE

Theorem 1 (FLT). For all $n \in \{3, 4, 5, \dots\}$ and for all $A, B, C \in \{1, 2, 3, \dots\}$: $A^n + B^n \neq C^n$.

Conjecture 1. For some $x, y, z \in \{3, 4, 5, \dots\}$ and for some $A, B, C \in \{1, 2, 3, \dots\}$ and for some prime number (a common prime factor) $\mathbf{p} \geq 2$:

$$(A^x + B^y = C^z \wedge \mathbf{p} \mid A, B, C).$$

Or - For all $x, y, z \in \{3, 4, 5, \dots\}$ and for all coprime $A, B, C \in \{1, 2, 3, \dots\}$:

$$(A^x + B^y \neq C^z).$$

This is The Beal's Conjecture (slightly restated).

III. THE TRULY MARVELLOUS PROOF OF FLT

Proof. Every even number which is not the power of number 2 has odd prime divisor, hence sufficient that we prove FLT for $n = 4$ and for odd prime numbers $n \in \mathbb{P}$.

Suppose that for $(n = 4 \vee n \in \mathbb{P})$ and for some coprime $A, B, C \in \{1, 2, 3, \dots\}$:

$$(A^n + B^n = C^n \wedge A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1}),$$

otherwise $A^n + B^n < C^n$.

Then only one number out of (A, B, C) is even and the even number $A + B - C > 0$ and for $n = 4$ the number C is odd in view of (1).

Without loss for this proof we can assume that $A, C - B \in \{1, 3, 5, \dots\}$ and $4 \nmid B, C$.

A. The Proof For $n = 4$.

For some coprime $B, C, A \in \{1, 2, 3, \dots\}$ such that $C, C - B, A \in \{1, 3, 5, \dots\}$ and $B < C > A$:

$$[B - (C - A) = 2v \wedge (C - A + 2v)^4 = B^4 = (C - A + A)^4 - A^4] \Rightarrow$$

$$(C - A)^2 2v + 6(C - A)v^2 + 8v^3 + \frac{4v^4}{C - A} = (C - A)^2 A + \frac{3}{2}(C - A)A^2 + A^3.$$

Thus for some $h, v, B, C, A \in \{1, 3, 5, \dots\}$:

$$(4h^4 + 2v = B \wedge 4h^4 = C - A \wedge h \mid v).$$

By analogy we get - for some $c, h, A, C \in \{1, 3, 5, \dots\}$:

$$(c^4 + 2ch = A \wedge ch = v \wedge c^4 = C - B).$$

Hence for some relatively prime $c, h \in \{1, 3, 5, \dots\}$:

$$(2ch + 4h^4)^4 = [(4h^4 + A)^2]^2 - (A^2)^2 = [(4h^4 + A)^2 + A^2](2h^4 + A)8h^4,$$

whence it implies that for some $z, w, x \in \{1, 3, 5, \dots\}$ and for some $y \in \{6, 10, 14, \dots\}$:

$$[zw = c + 2h^3 \wedge x + y = 2h^4 + A + 2h^4 \wedge x = 2h^4 + A \wedge y = 2h^4 \wedge$$

$$4 \nmid y \wedge 2(zw)^4 = ((x + y)^2 + (x - y)^2)x = 2(x^2 + y^2)x \wedge$$

$$z^4 w^4 = (x^2 + y^2)x \wedge (z^2)^2 = (x^2 + y^2) \wedge w^4 = x] \Rightarrow 4 \mid y,$$

which is inconsistent with $4 \nmid y$. This is the proof for $n = 4$. ♠

B. The Proof For $n \in \mathbb{P}$. General Deductions.

For some $n \in \mathbb{P}$ and some coprime $A, C, B \in \{1, 2, 3, \dots\}$ such that $A, C - B \in \{1, 3, 5, \dots\}$:

$$[A - (C - B) = B - (C - A) = 2v \wedge C - B + 2v = A \wedge C - A + 2v = B \wedge$$

$$(C - B + 2v)^n = (C - B + B)^n - B^n \wedge (C - A + 2v)^n = (C - A + A)^n - A^n \wedge$$

$$(A + B - B)^n + B^n = (A + B - 2v)^n = C^n].$$

Hence for some coprime $C, B, A \in \{1, 2, 3, \dots\}$ such that $C - B, A \in \{1, 3, 5, \dots\}$:

$$\{(C - B)^{n-2}v + (n - 1)(C - B)^{n-3}v^2 + \dots + 2^{n-2}v^{n-1} + \frac{2^{n-1}v^n}{n(C - B)} =$$

$$\frac{B}{2} \left[(C - B)^{n-2} + \frac{n - 1}{2}(C - B)^{n-3}B + \dots + B^{n-2} \right] \wedge$$

$$\begin{aligned}
& (C - A)^{n-2}2v + \frac{n-1}{2}(C - A)^{n-3}(2v)^2 + \dots + (2v)^{n-1} + \frac{(2v)^n}{n(C - A)} = \\
& A \left[(C - A)^{n-2} + \frac{n-1}{2}(C - A)^{n-3}A + \dots + A^{n-2} \right] \wedge \\
& (A + B)^{n-2}(-B) + \frac{n-1}{2}(A + B)^{n-3}(-B)^2 + \dots + (-B)^{n-1} = \\
& (A + B)^{n-2}(-2v) + \frac{n-1}{2}(A + B)^{n-3}(-2v)^2 + \dots + (-2v)^{n-1} + \frac{(-2v)^n}{n(A + B)}. \quad [5]
\end{aligned}$$

Thus for some $n \in \mathbb{P}$ and for some $m, c, h, A, C - B \in \{1, 3, 5, \dots\}$:

$$\begin{aligned}
& [nmch = v \wedge n \nmid mch \wedge n, C - A, A + B, C - B \mid (2v)^n \wedge \\
& (n \mid A, C - B \vee n \mid B, C - A \vee n \mid A + B, C) \wedge (4 \nmid B \wedge 4 \nmid C) \equiv 4 \nmid B, C].
\end{aligned}$$

B.1. The Proof For Odd $A, B, C - B$.

For some $n \in \mathbb{P}$ and some $m, c, h \in \{1, 3, 5, \dots\}$ with $n \nmid mch$:

$$\begin{aligned}
& [(n^{n-1}c^n + 2nmch = A \wedge n \mid A \wedge h^n + 2nmch = B \wedge \\
& 2^n m^n = A + B = n^{n-1}c^n + h^n + 4nmch \wedge n^{n-1}c^n + B = C) \vee \\
& (c^n + 2nmch = A \wedge n \mid B \wedge n^{n-1}h^n + 2nmch = B \wedge \\
& 2^n m^n = A + B = c^n + n^{n-1}h^n + 4nmch \wedge c^n + B = C) \vee \\
& (c^n + 2nmch = A \wedge n \mid A + B, C \wedge h^n + 2nmch = B \wedge \\
& 2^n n^{n-1}m^n = A + B = c^n + h^n + 4nmch \wedge c^n + B = C)] \Rightarrow \\
& [(2^n m^n - h^n = n^{n-1}c^n + 4nmch \wedge n \mid 2m - h \wedge n^2 \mid 2^n m^n - h^n) \vee \\
& (2^n m^n - c^n = n^{n-1}h^n + 4nmch \wedge n \mid 2m - c \wedge n^2 \mid 2^n m^n - c^n) \vee \\
& (2^n n^{n-1}m^n = c^n + h^n + 4nmch \wedge n \mid c + h \wedge n^2 \mid c^n + h^n)] \Rightarrow n \mid mch,
\end{aligned}$$

which is inconsistent with $n \nmid mch$. \spadesuit

B.2. The Proof For Even $B, C - A$.

For some $n \in \mathbb{P}$ and some $m, c, h \in \{1, 3, 5, \dots\}$ with $n \nmid mch$:

$$\begin{aligned}
& [(n^{n-1}c^n + 2nmch = A \wedge n \mid A \wedge 2^n h^n + 2nmch = B \wedge \\
& m^n = A + B = n^{n-1}c^n + 2^n h^n + 4nmch \wedge n^{n-1}c^n + B = C) \vee
\end{aligned}$$

$$\begin{aligned}
& (c^n + 2nmch = A \wedge n | B \wedge 2^n n^{n-1} h^n + 2nmch = B \wedge \\
& m^n = A + B = c^n + 2^n n^{n-1} h^n + 4nmch \wedge c^n + B = C) \vee \\
& (c^n + 2nmch = A \wedge n | A + B, C \wedge 2^n h^n + 2nmch = B \wedge \\
& n^{n-1} m^n = A + B = c^n + 2^n h^n + 4nmch \wedge c^n + B = C) \Rightarrow \\
& [(m^n - 2^n h^n = n^{n-1} c^n + 4nmch \wedge n | m - 2h \wedge n^2 | m^n - 2^n h^n) \vee \\
& (m^n - c^n = 2^n n^{n-1} h^n + 4nmch \wedge n | m - c \wedge n^2 | m^n - c^n) \vee \\
& (n^{n-1} m^n = c^n + 2^n h^n + 4nmch \wedge n | c + 2h \wedge n^2 | c^n + 2^n h^n)] \Rightarrow n | mch,
\end{aligned}$$

which is inconsistent with $n \nmid mch$. This is the proof.

IV. TWO FALSE PROOF OF FLT FOR $n = 4$ WITH $4 | B$

False Proof 1. For some $u, v \in \{1, 2, 3, \dots\}$ and for some $A, C, a, k, b, c \in \{1, 3, 5, \dots\}$ and for some $B \in \{6, 10, 14, \dots\}$ such that $u - v \in \{1, 3, 5, \dots\}$ and $k > 1$ and $C > a > 2^{\frac{k}{2}} bc$ and u, v, A, C are coprime and A, C, a, b, c are coprime:

$$\begin{aligned}
& \{[u^2 - v^2 = A^2 \wedge u^2 + v^2 = C^2 \wedge u = a^2 \wedge v = 2^k b^2 c^2 \wedge 2uv = B^2 \wedge \\
& 2u^2 = C^2 + A^2 \wedge a^4 + \left(2^{\frac{k}{2}} bc\right)^4 = C^2 \equiv 0 \wedge (A^2)^2 + (B^2)^2 = (C^2)^2] \in \mathbf{0}\},
\end{aligned}$$

inasmuch as $2^{\frac{k}{2}} \notin \{2, 4, 8, \dots\}$.

Moreover on the strength of (1):

$$(C = A + v \wedge \pm A = A - v) \Rightarrow [(v = 0 \vee v = 2A) \wedge \mathbf{gcd}(v, A) > 1],$$

which is inconsistent with $[v > 0 \wedge \mathbf{gcd}(v, A) = 1]$. This is the false proof 1.

False Proof 2. For some relatively prime $u, v \in \{1, 2, 3, \dots\}$ and for some $C \in \{1, 3, 5, \dots\}$ and for some $B \in \{6, 10, 14, \dots\}$ such that $u + v, u - v, C$, and B are coprime:

$$\begin{aligned}
& \{[u + v = p \wedge u - v = q \wedge (u^2 - v^2)^2 = (u + v)^2 (u - v)^2 = (pq)^2 = \\
& (u^2 + v^2)^2 - (2uv)^2 = (C^2)^2 - (B^2)^2 \wedge u^2 + v^2 = C^2 \wedge 2uv = B^2 \wedge \\
& (C^2 + B^2 = p^2 \wedge 4 | B \wedge C^2 - B^2 = q^2 \quad [2] \text{ or } [7])] \in \mathbf{0}\},
\end{aligned}$$

in view of the false proof 1 - also.

Further Weil's assumed that for some relatively prime $r, s \in \{1, 3, 5, \dots\}$:

$$\left[(2rs\sqrt{2})^2 = 2B^2 = p^2 - q^2 \wedge p = r^2 + 2s^2 \wedge \pm q = r^2 - 2s^2 [7] \right].$$

But this sentence cannot exist because

$$(2rs\sqrt{2})^2 = 2B^2 \Rightarrow (2rs)^2 = B^2 \Rightarrow 2rs = B \Rightarrow 4 \nmid B,$$

which is inconsistent with $4 \mid B$. This is the false proof 2.

V. THE INCOMPLETE PROOF OF FLT FOR $n \in \mathbb{P}$

Incomplete Proof. We assume that for some $n \in \mathbb{P}$ and for some coprime $p, q, w, r, x \in \{1, 3, 5, \dots\}$ such that $p > q$ and $w > r$:

$$\{[n \mid pq \wedge (2pq)^n = B^n = (w^n r^n)^2 - (x^n)^2 \wedge n \nmid 2pq + x^2, wr, x \wedge$$

$$\frac{(2pq)^n + (2^{n-1} q^n)^2}{2(2^{n-1} q^n)} = p^n + 2^{n-2} q^n = (wr)^n = C^{\frac{n}{2}} \wedge (wr)^2 = C \wedge$$

$$\frac{(2pq)^n - (2^{n-1} q^n)^2}{2(2^{n-1} q^n)} = p^n - 2^{n-2} q^n = x^n = A^{\frac{n}{2}} \wedge x^2 = A \wedge$$

$$\frac{(2pq)^n + (x^2)^n}{2pq + x^2} = \frac{(2pq)^n + (x^2)^n}{(r^2)^n} = \frac{(w^2 r^2)^n}{(r^2)^n} = (w^2)^n \wedge$$

$$(r^n)^2 - x^2 = 2pq \wedge (2 \mid pq \equiv 0)] \in \mathbf{0}\}.$$

The proof is incomplete because does not include the case for $C \in \{6, 10, 14, \dots\}$ and does not include the case for $C, A \in \{3, 5, 7, \dots\} \setminus \{3^2, 5^2, 7^2, \dots\}$. This is the incomplete proof.

VI. THE PROOF OF THE BEAL'S CONJECTURE

Lemma 1. For all $n \in \{3, 5, 7, \dots\}$ and for all $a \in \{2, 4, 6, \dots\}$ and for all $b \in \{1, 3, 5, \dots\}$ and for some $c \in \{1, 3, 5, \dots\}$ and for some $\mathbf{p} > 2$ with $a > b$ and $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n - b^n}{a - b} = c \wedge \mathbf{gcd}(a, (a - b)c) = \mathbf{gcd}(b, (a - b)c) = 1 \right] \Rightarrow$$

$$(a(a - b)c)^n - (b(a - b)c)^n = ((a - b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\mathbf{p} \mid (a - b)c, A, C, B.$$

This is the lemma 1.

Example 1.

$$8^3 - 7^3 = 13^2 \Rightarrow 1352^3 - 1183^3 = 169^4 = 13^8 \Rightarrow 104^3 - 91^3 = 13^5.$$

This is the example 1.

Lemma 2. For all $n \in \{3,5,7, \dots\}$ and for all $a \in \{1,3,5, \dots\}$ and for all $b \in \{2,4,6, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} > 2$ with $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n + b^n}{a + b} = c \wedge \mathbf{gcd}(a, (a + b)c) = \mathbf{gcd}(b, (a + b)c) = 1 \right] \Rightarrow$$

$$(a(a + b)c)^n + (b(a + b)c)^n = ((a + b)c)^{n+1} = C^{n+1} = A^n + B^n \Rightarrow$$

$$\mathbf{p} \mid (a + b)c, C, A, B.$$

This is the lemma 2.

Example 2.

$$1^3 + 2^3 = 3(1 + 2) = 3^2 \Rightarrow 9^3 + 18^3 = 9^4 = 3^8 \Rightarrow (3^3 + 6^3 = 3^{3+2} [3]).$$

This is the example 2.

Lemma 3. For all $n \in \{3,5,7, \dots\}$ and for all $a, b \in \{1,3,5, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} \geq 2$ with $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n + b^n}{a + b} = c \wedge \mathbf{gcd}(a, (a + b)c) = \mathbf{gcd}(b, (a + b)c) = 1 \right] \Rightarrow$$

$$(a(a + b)c)^n + (b(a + b)c)^n = ((a + b)c)^{n+1} = C^{n+1} = A^n + B^n \Rightarrow$$

$$\mathbf{p} \mid (a + b)c, C, A, B.$$

This is the lemma 3.

Example 3.

$$1^n + 1^n = 2 \Rightarrow (2^n + 2^n = 2^{n+1} [3]).$$

This is the example 3.

Lemma 4. For all $n \in \{3,5,7, \dots\}$ and for all $a, b \in \{1,3,5, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} \geq 2$ with $a > b$ and $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n - b^n}{a - b} = c \wedge \mathbf{gcd}(a, (a - b)c) = \mathbf{gcd}(b, (a - b)c) = 1 \right] \Rightarrow$$

$$(a(a - b)c)^n - (b(a - b)c)^n = ((a - b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\mathbf{p} \mid (a - b)c, A, C, B.$$

This is the lemma 4.

Lemma 5. For all $n, a \in \{3,5,7, \dots\}$ and for all $b \in \{2,4,6, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} > 2$ with $a > b$ and $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n - b^n}{a - b} = c \wedge \mathbf{gcd}(a, (a - b)c) = \mathbf{gcd}(b, (a - b)c) = 1 \right] \Rightarrow$$

$$(a(a - b)c)^n - (b(a - b)c)^n = ((a - b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\mathbf{p} \mid (a - b)c, A, C, B.$$

This is the lemma 5.

Lemma 6. For all $n \in \{4,6,8, \dots\}$ and for all $a, b \in \{1,3,5, \dots\}$ and for some $c \in \{2,4,6, \dots\}$ and for some $\mathbf{p} \geq 2$ with $a > b$ and $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n - b^n}{a - b} = c \wedge \mathbf{gcd}(a, (a - b)c) = \mathbf{gcd}(b, (a - b)c) = 1 \right] \Rightarrow$$

$$(a(a - b)c)^n - (b(a - b)c)^n = ((a - b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\mathbf{p} \mid (a - b)c, A, C, B.$$

This is the lemma 6.

Lemma 7. For all $n \in \{4,6,8, \dots\}$ and for all $a \in \{2,4,6, \dots\}$ and for all $b \in \{1,3,5, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} > 2$ with $a > b$ and $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n - b^n}{a - b} = c \wedge \mathbf{gcd}(a, (a - b)c) = \mathbf{gcd}(b, (a - b)c) = 1 \right] \Rightarrow$$

$$(a(a - b)c)^n - (b(a - b)c)^n = ((a - b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\mathbf{p} \mid (a - b)c, A, C, B.$$

This is the lemma 7.

Corollary 1. For all $n \in \{3,4,5, \dots\}$ and for all $a \in \{2,3,4, \dots\}$ and for some $c \in \{1,2,3, \dots\}$:

$$(a(a^n - 1))^n - (a^n - 1)^n = (a^n - 1)^{n+1} =$$

$$(ac(a - 1))^n - (c(a - 1))^n = (c(a - 1))^{n+1} \Rightarrow$$

$$[(a^n - 1)^{2n} + (a^n - 1)^{2n+1} = (a(a^n - 1)^2)^n \quad [3]].$$

This is the corollary 1.

Lemma 8. For all $n \in \{4,6,8, \dots\}$ and for all $a \in \{3,5,7, \dots\}$ and for all $b \in \{2,4,6, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} > 2$ with $a > b$ and $\mathbf{gcd}(a, b) = 1$:

$$\left[\frac{a^n - b^n}{a - b} = c \wedge \mathbf{gcd}(a, (a - b)c) = \mathbf{gcd}(b, (a - b)c) = 1 \right] \Rightarrow$$

$$(a(a-b)c)^n - (b(a-b)c)^n = ((a-b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\mathbf{p} \mid (a-b)c, A, C, B.$$

This is the lemma 8.

Lemma 9. For all $n \in \{4,6,8, \dots\}$ and for all $a \in \{1,3,5, \dots\}$ and for all $b \in \{2,4,6, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} > 2$ with $\mathbf{gcd}(a, b) = 1$:

$$[a^n + b^n = c \wedge \mathbf{gcd}(a, c) = \mathbf{gcd}(b, c) = 1] \Rightarrow$$

$$(ac)^n + (bc)^n = c^{n+1} = C^{n+1} = A^n + B^n \Rightarrow \mathbf{p} \mid a^n + b^n, C, A, B.$$

This is the lemma 9.

Lemma 10. For all $n \in \{4,6,8, \dots\}$ and for all $a, b \in \{1,3,5, \dots\}$ and for some $c \in \{1,3,5, \dots\}$ and for some $\mathbf{p} \geq 2$ with $\mathbf{gcd}(a, b) = 1$:

$$[a^n + b^n = 2c \wedge \mathbf{gcd}(a, c) = \mathbf{gcd}(b, c) = 1] \Rightarrow$$

$$(2ac)^n + (2bc)^n = (2c)^{n+1} = C^{n+1} = A^n + B^n \Rightarrow \mathbf{p} \mid 2c, C, A, B.$$

This is the lemma 10.

Proof of Beal Conjecture. The above lemmas gives always one of the below equations

$$[(A^{n+1} + B^n = C^n \equiv A^x + B^y = C^z) \vee (A^n + B^n = C^{n+1} \equiv A^x + B^y = C^z)],$$

where $n, x, y, z \in \{3,4,5, \dots\}$ and A, B and C have the common prime factor $\mathbf{p} \geq 2$.

These lemmas gives all solutions of the equation $A^x + B^y = C^z$ such that A, B and C have the common prime factor $\mathbf{p} \geq 2$, and the number of the solutions $[A, B, C]$ is infinite. This is the proof.

Theorem 2 (Beal Theorem). On the strength of the proof of the Main Conjecture –

For all $x, y, z \in \{3,4,5, \dots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions in $\{1,2,3, \dots\}$.

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