THE PROOF OF THE ANDY BEAL'S CONJECTURE

LESZEK W. GUŁA

Dedicated to my Parents and my Brother

Abstract. 1. The truly marvellous proof of The Fermat's Last Theorem (FLT). 2. Three false proofs of FLT for $n = 4$. 3. The false and incomplete proof of FLT for $n \in \mathbb{P}$. 4. The proof of The Andy Beal's Conjecture.

I. Introduction

The cover of this issue of the Bulletin is the frontispiece to a volume of Samuel de Fermat's 1670 edition of Bachet's Latin translation of Diophantus's Arithmetica. This edition includes the marginalia of the editor's father, Pierre de Fermat. Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation $x^2 + y^2 = z^2$, the marginal comment that hints at the existence of a proof (a demonstratio sane mirabilis) of what has come to be known as Fermat's Last Theorem. Diophantus's work had fired the imagination of the Italian Renaissance mathematician Rafael Bombelli, as it inspired Fermat a century later. [6]

Problem II.8 of the *Diophantus's Arithmetica* asks how a given square number is split into two other squares. Diophantus's shows how to solve this sum-of-squares problem for $k = 4$ and $u = 2$ [8] , inasmuch as for all $k, u \in \mathbb{Z}$:

$$k^2 = \left(\frac{2ku}{u^2+1}\right)^2 + \left[\frac{k\left(u^2-1\right)}{u^2+1}\right]^2 . \text{ [4]}$$

Thus for all relatively prime natural numbers u, v such that $u - v \in \{1, 3, 5, ...\}$:

$$\left(u^2+v^2\right)^2 = u^4 - 2u^2v^2 + v^4 + 4u^2v^2 = \left(u^2-v^2\right)^2 + (2uv)^2 .$$

We have a primitive Pythagorean triple $\left(u^2 - v^2,\ 2uv,\ u^2 + v^2\right) = (x, y, z)$ – the primitive triple because the numbers x, y, and z are co-prime.

Around 1637, Fermat wrote his Last Theorem in the margin of his copy of the *Arithmetica* next to Diophantus sum-of-squares problem: it is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain. In number theory, Fermat's Last Theorem (FLT) states that no three positive integers A, B, and C can satisfy the equation $A^n + B^n = C^n$ for any integer value of n greater than two. [8]

Date: 03 March 1994 – 12 June 2013 – 28 August 2014 – 01 - 02 - 05 - 09 - 25 October 2014.
1991 *Mathematics Subject Classification.* Primary: 11D41, 11D45; Secondary: 11D85.
Key words and phrases. Common Prime Factor, Diophantine Equations, Fermat Equation, Greatest Common Divisor, Newton Binomial Formula.

It is easy to see that if $A^n + B^n = C^n$ then either A, B, and C are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor). You could then restate FLT by saying that $A^n + B^n = C^n$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [1]

It is known that for some co-prime $x, y, z \in \{3, 4, 5, ...\}$:

$$x^2 + y^2 = z^2 \wedge \ (x+y)^2 + (x-y)^2 = 2z^2, \tag{1}$$

where z is odd because for all $a, b \in \mathbb{N}$ the number $\dfrac{(2a+1)^2 + (2b+1)^2}{2}$ is odd.

II. The Andy Beal's Conjecture

Conjecture 1. *For some* $x,\ y,\ z \in \mathbb{N}_3$ *and for some* $A,\ B,\ C \in \mathbb{N}_1$ *and for some prime number (a common prime factor)* $\mathbf{p} \geq 2$*:*

$$(A^x + B^y = C^z \wedge \ \mathbf{p} \mid A,\ B,\ C).$$

Or – For all $x,\ y,\ z \in \mathbb{N}_3$ *and for all co-prime* $A,\ B,\ C \in \mathbb{N}_1$*:*

$$A^x + B^y \neq C^z.$$

This is The Andy Beal's Conjecture (slightly restated).

III. The Truly Marvellous Proof Of FLT For $n \in \mathbb{N}_3$

Every even number which is not the power of number 2, has odd prime divisor, hence sufficient that we prove FLT for $n = 4$ and for odd prime numbers $n \in \mathbb{P}$.

Proof of FLT. Suppose that for $n = 4$ or for some $n \in \mathbb{P}$, and for some coprime numbers $A,\ B,\ C \in \mathbb{N}_1$:

(2) $A^n + B^n = C^n \wedge A + B > C \wedge \ A^2 + B^2 > C^2 \wedge \ldots \wedge \ A^{n-1} + B^{n-1} > Z^{n-1}$,

otherwise $X^n + Y^n - Z^n < 0$.

From (2) it follows that only one number out of (A, B, C) is even and the even number $A + B - C > 0$, and if $n = 4$, then C is odd in view of (1). Moreover we assume that the numbers $A,\ C - B$ are odd.

A. The Proof For $n = 4$.

For some $\nu \in \mathbb{N}_1$ and for some coprime $C,\ A,\ B \in \mathbb{N}_1$ such that $C,\ A,\ C - B \in \{1, 3, 5, ...\}$:

$$\left[2\nu = B - (C - A) \wedge \ (C - A + 2\nu)^4 = B^4 = (C - A + A)^4 - A^4\right] \Rightarrow$$

$$(C-A)^2\, 2\nu + 6\,(C-A)\,\nu^2 + 8\nu^3 + \frac{4\nu^4}{C-A} = (C-A)^2 A + \frac{3}{2}\,(C-A)\,A^2 + A^3.$$

Thus for some $h,\ \nu,\ C,\ A \in \{1, 3, 5, ...\}$:

$$\left(4h^4 + 2\nu = B \wedge \ 4h^4 = C - A \wedge \ h \mid \nu \wedge \ 4 \nmid B\right).$$

Equally – for some relatively prime $u,\,v\in\mathbb{N}_1$ such that $u,\,u-v\in\{1,3,5,...\}$:

$$\left(2uv=B^2\wedge\ u^2+v^2=C^2\wedge\ u^2-v^2=A^2\wedge\ 4\mid v,\ B\right),$$

which is inconsistent with $4\nmid B$. This is the proof. ✠

B. The Proof For $n\in\mathbb{P}$. General Deductions.

For some $\nu\in\mathbb{N}_1$ and for some $A,\,C-B\in\{1,3,5,...\}$:

$$2\nu=A-(C-B)=B-(C-A)\wedge\ C-B+2\nu=A\wedge\ C-A+2\nu=B\ \wedge$$

$$(C-B+2\nu)^n=(C-B+B)^n-B^n\wedge\ (C-A+2\nu)^n=(C-A+A)^n-A^n\ \wedge$$

$$[A+B+(-B)]^n+B^n=[A+B+(-2\nu)]^n=C^n.$$

Consequently we get

$$(C-B)^{n-2}\nu+(n-1)(C-B)^{n-3}\nu^2+...+2^{n-2}\nu^{n-1}+\frac{2^{n-1}\nu^n}{n(C-B)}=$$

$$\frac{B}{2}\left[(C-B)^{n-2}+\frac{n-1}{2}(C-B)^{n-3}B+...+B^{n-2}\right]\ \wedge$$

$$(C-A)^{n-2}2\nu+\frac{n-1}{2}(C-A)^{n-3}(2\nu)^2+...+(2\nu)^{n-1}+\frac{(2\nu)^n}{n(C-A)}=$$

$$A\left[(C-A)^{n-2}+\frac{n-1}{2}(C-A)^{n-3}A+...+A^{n-2}\right]\ \wedge$$

$$(A+B)^{n-2}(-B)+\frac{n-1}{2}(A+B)^{n-3}(-B)^2+...+(-B)^{n-1}=$$

$$(A+B)^{n-2}(-2\nu)+\frac{n-1}{2}(A+B)^{n-3}(-2\nu)^2+...+(-2\nu)^{n-1}+\frac{(-2\nu)^n}{n(A+B)}.\quad [5]$$

Hence for some $n\in\mathbb{P}$ and for some $m,\,c,\,h,\,A,\,C-B\in\{1,3,5,...\}$:

$$[nmch=\nu\wedge\ n\nmid mch\wedge\ n,\ C-A,\ A+B,\ C-B\mid(2\nu)^n\ \wedge$$

$$(n\mid A,\ C-B\ \vee\ n\mid B,\ C-A\ \vee\ n\mid A+B,\ C)\wedge\ (4\nmid B\vee 4\nmid C)].$$

B.1. Proof For Odd $A,\,B,\,C-B$.

For some $m,\,c,\,h\in\{1,3,5,...\}$:

$$\{n\nmid mch\wedge\ \left[\left(n^{n-1}c^n+2nmch=A\wedge\ n\mid A\wedge\ h^n+2nmch=B\ \wedge\right.\right.$$

$$\left.2^nm^n=A+B=n^{n-1}c^n+h^n+4nmch\wedge\ n^{n-1}c^n+B=C\right)\ \vee$$

$$\left(c^n+2nmch=A\wedge\ n\mid B\wedge\ n^{n-1}h^n+2nmch=B\ \wedge\right.$$

$$\left.2^nm^n=A+B=c^n+n^{n-1}h^n+4nmch\wedge\ c^n+B=C\right)\ \vee$$

$$\left(c^n+2nmch=A\wedge\ n\mid A+B,\ C\wedge\ h^n+2nmch=B\ \wedge\right.$$

$$\left.\left.2^nn^{n-1}m^n=A+B=c^n+h^n+4nmch\wedge\ c^n+B=C\right)\right]\}\Rightarrow$$

$$\left[\left(2^nm^n-h^n=n^{n-1}c^n+4nmch\wedge\ n\mid 2m-h\wedge\ n^2\mid 2^nm^n-h^n\right)\ \vee\right.$$

$$\left(2^nm^n-c^n=n^{n-1}h^n+4nmch\wedge\ n\mid 2m-c\wedge\ n^2\mid 2^nm^n-c^n\right)\ \vee$$

$$\left.\left(2^nn^{n-1}m^n=c^n+h^n+4nmch\wedge\ n\mid c+h\wedge\ n^2\mid c^n+h^n\right)\right]\Rightarrow n\mid mch,$$

which is isconsistent with $n\nmid mch$. ✠

B.2. Proof For Even $B, C-A$.

For some $m,\, c,\, h \in \{1,3,5,...\}$:

$$\{n \nmid mch \wedge \ \big[\big(n^{n-1}c^n + 2nmch = A \wedge \ n \mid A \wedge \ 2^n h^n + 2nmch = B \ \wedge$$
$$m^n = A + B = n^{n-1}c^n + 2^n h^n + 4nmch \wedge \ n^{n-1}c^n + B = C\big) \ \vee$$
$$\big(c^n + 2nmch = A \wedge \ n \mid B \wedge \ 2^n n^{n-1} h^n + 2nmch = B \ \wedge$$
$$m^n = A + B = c^n + 2^n n^{n-1} h^n + 4nmch \wedge \ c^n + B = C\big) \ \vee$$
$$(c^n + 2nmch = A \wedge \ n \mid A + B,\ C \ \wedge \ 2^n h^n + 2nmch = B \ \wedge$$
$$n^{n-1}m^n = A + B = c^n + 2^n h^n + 4nmch \wedge \ c^n + B = C)\big]\big\} \Rightarrow$$
$$\big[\big(m^n - 2^n h^n = n^{n-1}c^n + 4nmch \wedge \ n \mid m - 2h \wedge \ n^2 \mid m^n - 2^n h^n\big) \ \vee$$
$$\big(m^n - c^n = 2^n n^{n-1} h^n + 4nmch \wedge \ n \mid m - c \ \wedge \ n^2 \mid m^n - c^n\big) \ \vee$$
$$\big(n^{n-1}m^n = c^n + 2^n h^n + 4nmch \wedge \ n \mid c + 2h \ \wedge \ n^2 \mid c^n + 2^n h^n\big)\big] \Rightarrow n \mid mch,$$

which is inconsistent with $n \nmid mch$. This is the proof. □

III. Three False Proofs Of FLT For $n = 4$ With $4 \nmid B$

False Proof 1. Suppose that for some relatively prime $u,\, v \in \mathbb{N}_1$ and for some coprime $A,\, B,\, C \in \mathbb{N}_1$ such that $u,\, u - v \in \{1,3,5,...\}$ and $B \in \{6,10,14,...\}$:

$$\big\{\big[\big(u^2 - v^2 = A^2 \wedge \ 2uv = B^2 \wedge \ 4 \mid v,\ B\big) \equiv 0 \wedge \ u^2 + v^2 = C^2 \ \wedge$$
$$\left(A^2\right)^2 + \left(B^2\right)^2 = \left(C^2\right)^2 \wedge \ \gcd(v, A) = 1\big] \in \mathbf{0}\big\}.$$

Hence for some $a,\, k,\, b,\, C \in \{1,3,5,...\}$ such that $a,\, b,\, C > 1$ and $C > a > 2^{\frac{k}{2}} b$ and $\gcd(a,b) = \gcd(a,C) = \gcd(b,C) = 1$:

$$\left\{\left[a^2 = u \wedge \ 2^k b^2 = v \wedge \ a^4 + \left(2^{\frac{k}{2}} b\right)^4 = C^2 \equiv 0\right] \in \mathbf{0}\right\}$$

because $2^{\frac{k}{2}} \notin \{2,4,8,...\}$.

Moreover on the strength of (1):

$$\big(u^2 = A^2 + v^2 \wedge \ 2u^2 = C^2 + A^2\big) \Rightarrow (C = A + v \wedge \ \pm A = A - v) \Rightarrow$$
$$[v = 0 \vee (v = 2A \wedge \ \gcd(v, A) > 1)],$$

which is inconsistent with $[v > 0 \wedge \ \gcd(v, A) = 1]$. This is the false proof 1. □

Corollary 1. *The equation*

$$A^4 + B^4 = C^4$$

has no primitive solutions $[A, B, C]$ *in* $\mathbb{N}_1$, *where for some relatively prime natural numbers* $u > v$: $A^2 = u^2 - v^2 = (u+v)(u-v) = pq$ *or* $B^2 = 2uv$ *or* $C^2 = u^2 + v^2$.

This is the corollary 1.

Example 1. *For all relatively prime natural numbers* $u,\ v$ *such that* $u - v \in \{1, 3, 5, \ldots\}$ *and* $4 \nmid \sqrt{2uv}$ *the equation:*

$$\left(u^2 - v^2\right)^2 = \left(\sqrt{u^2 - v^2}\right)^4 = \left(\sqrt{u^2 + v^2}\right)^4 - \left(\sqrt{2uv}\right)^4$$

has no primitive solutions $\left[u^2 - v^2,\ \sqrt{u^2 + v^2},\ \sqrt{2uv}\right]$ *in* $\mathbb{N}_1$.

This is the example 1.

Proof of Example 1. Suppose that for some relatively prime natural numbers $u,\ v$ such that $u - v \in \{1, 3, 5, \ldots\}$ and $4 \nmid \sqrt{2uv}$ the equation:

$$\left(u^2 - v^2\right)^2 = \left(\sqrt{u^2 - v^2}\right)^4 = \left(\sqrt{u^2 + v^2}\right)^4 - \left(\sqrt{2uv}\right)^4$$

has primitive solutions $\left[u^2 - v^2,\ \sqrt{u^2 + v^2},\ \sqrt{2uv}\right]$ in $\mathbb{N}_1$.

Thus for some relatively prime natural numbers $u,\ v$ such that $u - v \in \{1, 3, 5, \ldots\}$ and $4 \nmid \sqrt{2uv}$:

$$\left[(u+v)(u-v)\right]^2 = (pq)^2 = \left(\sqrt{pq}\right)^4 = \left(\sqrt{u^2 + v^2}\right)^4 - \left(\sqrt{2uv}\right)^4 \Rightarrow$$

$$\left[\left(\sqrt{u^2 + v^2}\right)^2 + \left(\sqrt{2uv}\right)^2 = (u+v)^2 = p^2 \ \wedge\right.$$

$$\left.\left(\sqrt{u^2 + v^2}\right)^2 - \left(\sqrt{2uv}\right)^2 = (u-v)^2 = q^2\right] \Rightarrow 4 \mid \sqrt{2uv},$$

which is inconsistent with $4 \nmid \sqrt{2uv}$. This is the proof of the example 1. □

Corollary 2. *It is easy to verify that*

$$\left(u^2 - v^2\right)^2 = \left[(u+v)(u-v)\right]^2 = (pq)^2 = A^4 = \left(\sqrt{u^2 - v^2}\right)^4 =$$

$$\left(\sqrt{u^2 + v^2}\right)^4 - \left(\sqrt{2uv}\right)^4 = \left[\left(\sqrt{u^2 + v^2}\right)^2\right]^2 - \left[\left(\sqrt{2uv}\right)^2\right]^2 =$$

$$\left(u^2 + v^2\right)^2 - (2uv)^2 = (u+v)^2 (u-v)^2 = (pq)^2 = \left(C^2\right)^2 - \left(B^2\right)^2 =$$

$$\left(C^2 + B^2\right)\left(C^2 - B^2\right) \wedge \ \left(C^2 + B^2 = p^2 \wedge\ 4 \mid B \wedge\ C^2 - B^2 = q^2 \text{ [2] or [7] }\right),$$

which is inconsistent with $4 \nmid B$. ✠

This is the corollary 2.

False Proof 2. We have – for some co-prime $C,\ B,\ p,\ q \in \mathbb{N}_1$, where only B is even and $4 \nmid B$:

$$\left[\left(C^2 + B^2 = p^2 \wedge\ 4 \mid B \wedge\ C^2 - B^2 = q^2 \text{ [2] or [7] }\right) \wedge\ C^4 - B^4 = (pq)^2 = A^4\right],$$

which is inconsistent with $4 \nmid B$. ✠

Further Weil's assumed that for some relatively prime odd natural numbers $r,\ s$:

$$\left[\left(2rs\sqrt{2}\right)^2 = 2B^2 = p^2 - q^2 \wedge\ p = r^2 + 2s^2 \wedge\ \pm q = r^2 - 2s^2 \text{ [7] }\right].$$

But this sentence cannot exists because

$$\left(2rs\sqrt{2}\right)^2 = 2B^2 \Rightarrow 2\left(2rs\right)^2 = 2B^2 \Rightarrow 2rs = B \Rightarrow 4 \nmid B,$$

which is inconsistent with $4 \mid B$.

Moreover on the strength of (1) we get:

$$\left[\left(C^2 = B^2 + q^2 \equiv 0\right) \wedge\ 2C^2 = p^2 + q^2 \wedge\ p = B + q \wedge\ \pm q = B - q\right] \Rightarrow$$
$$\left[\left(B = 2q \wedge\ \gcd\left(q, B\right) > 1\right) \vee B = 0\right],$$

which is inconsistent with $[\gcd\left(q, B\right) = 1 \wedge\ B > 0]$. This is the false proof 2. □

Remark 1. *Below we have the hypothetical solutions of the equations* $\left(pq\right)^2 = C^4 - B^4$ *or* $\left(pq\right)^2 = A^4 = C^4 - B^4$*:*

$$\left[u^2 - v^2,\ \sqrt{u^2+v^2},\ \sqrt{2uv}\right] = \left[u^2 - v^2,\ C,\ B\right] = \left[pq,\ \sqrt{u^2+v^2},\ \sqrt{2uv}\right]$$

or

$$\left[\sqrt{u^2-v^2},\ \sqrt{u^2+v^2},\ \sqrt{2uv}\right] = \left[\sqrt{u^2-v^2},\ C,\ B\right] = \left[\sqrt{pq},\ \sqrt{u^2+v^2},\ \sqrt{2uv}\right].$$

$C^2 = B^2 + q^2 \equiv 0$ *because* $C^2 = B^2 + q^2 \Rightarrow 4 \mid B$*, which is inconsistent with* $4 \nmid B$*.*

This is the remark 1.

Example 2. *For all coprime* $u,\ v,\ C,\ A \in \mathbb{N}_1$ *and for all* $B \in \{6, 10, 14, ...\}$ *such that* $u,\ u-v,\ C,\ A\ ,\ C-B$ *are odd the equation:*

$$\left(2uv\right)^2 = C^4 - A^4 = B^4$$

has no primitive solutions $\left[2uv,\ \sqrt{u^2+v^2},\ \sqrt{u^2-v^2}\right]$ *in* $\mathbb{N}_1$*.*

This is the example 2.

Proof of Example 2. Suppose that for some coprime $u,\ v,\ C,\ A \in \mathbb{N}_1$ and for some $B \in \{6, 10, 14, ...\}$ such that $u,\ u-v,\ C,\ A\ ,\ C-B$ are odd:

$$\left[u^2 + v^2 = C^2 \wedge\ u^2 - v^2 = A^2 \wedge\ \left(2uv\right)^2 = C^4 - A^4 = B^4 \wedge\ 4 \mid v,\ B\right],$$

which is inconsistent with $4 \nmid B$. This is the proof of the example 2. □

Remark 2. $\left[2uv,\ \sqrt{u^2+v^2},\ \sqrt{u^2-v^2}\right] = \left[2uv,\ C,\ A\right]$. *This is the remark 2.*

False Proof 3. For some coprime $u,\ v,\ C,\ A \in \mathbb{N}_1$ and for some $B \in \{6, 10, 14, ...\}$ such that $u,\ u-v,\ C,\ A\ ,\ C-B$ are odd:

$$\left[u^2 + v^2 = C^2 \wedge\ u^2 - v^2 = A^2 \wedge\ \left(2uv\right)^2 = B^4 = C^4 - A^4 \wedge\ 4 \mid v,\ B\right],$$

which is inconsistent with $4 \nmid B$. ✠

Further – on the strength of (1):

$$\left(u^2 = A^2 + v^2 \wedge\ 2u^2 = C^2 + A^2\right) \Rightarrow \left(C = A + v \wedge\ \pm A = A - v\right) \Rightarrow$$
$$\left[v = 0 \vee \left(v = 2A \wedge\ \gcd\left(v, A\right) > 1\right)\right],$$

which is inconsistent with $[v > 0 \wedge\ \gcd\left(v, A\right) = 1]$. This is the false proof 3. □

IV. The False And Incomplete Proof Of FLT For $n \in \mathbb{P}$

False And Incomplete Proof For $n \in \mathbb{P}$. On the strength of the above proof of FLT the Fermat Equation is false for all $n \in \mathbb{P}$ and for all $A,\, C,\, B \in \mathbb{N}_1$.

However we assume that for some $n \in \mathbb{P}$ and for some coprime odd natural numbers $p,\, q,\, w,\, r,\, x$ such that $p > q$ and $w > r$:

$$\Big\{\Big[n \mid pq \wedge\ (2pq)^n = B^n = (w^n r^n)^2 - (x^n)^2 \wedge\ n \nmid 2pq + x^2,\ wr,\ x\ \wedge$$

$$\frac{(2pq)^n + \left(2^{n-1}q^n\right)^2}{2\left(2^{n-1}q^n\right)} = p^n + 2^{n-2}q^n = (wr)^n = C^{\frac{n}{2}} \wedge\ (wr)^2 = C\ \wedge$$

$$\frac{(2pq)^n - \left(2^{n-1}q^n\right)^2}{2\left(2^{n-1}q^n\right)} = p^n - 2^{n-2}q^n = x^n = A^{\frac{n}{2}} \wedge\ x^2 = A\ \wedge$$

$$\frac{(2pq)^n + \left(x^2\right)^n}{2pq + x^2} = \frac{(2pq)^n + \left(x^2\right)^n}{\left(r^2\right)^n} = \frac{\left(w^2r^2\right)^n}{\left(r^2\right)^n} = \left(w^2\right)^n\ \wedge$$

$$(r^n)^2 - x^2 = 2pq \wedge\ (2 \mid pq \equiv 0)\Big] \in \mathbf{0}\Big\}.$$

The proof is incomplete because does not include the cases for $C \in \{6, 10, 14, ...\}$.

This is the false and incomplete proof for $n \in \mathbb{P}$. □

V. The Proof Of The Andy Beal's Conjecture

Lemma 1. *For all* $n \in \{3, 5, 7, ...\}$ *and for all* $a \in \{2, 4, 6, ...\}$ *and for all* $b \in \{1, 3, 5, ...\}$ *and for some* $c \in \{1, 3, 5, ...\}$ *and for some* $\mathbf{p} > 2$ *with* $a > b$ *and* $\gcd(a, b) = 1$*:*

$$\left\{\left[\frac{a^n - b^n}{a - b} = c \wedge\ \gcd(a, (a-b)\,c) = \gcd(b, (a-b)\,c) = 1\right] \Rightarrow\right.$$

$$(a\,(a-b)\,c)^n - (b\,(a-b)\,c)^n = ((a-b)\,c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\left.\mathbf{p} \mid (a-b)\,c,\ A,\ C,\ B\right\}.$$

This is the lemma 1.

Example 3.

$$8^3 - 7^3 = 13^2 \Rightarrow 1352^3 - 1183^3 = 169^4 = 13^8 \Rightarrow 104^3 - 91^3 = 13^5.$$

This is the example 3.

Lemma 2. *For all* $n \in \{3,5,7,...\}$ *and for all* $a \in \{1,3,5,...\}$ *and for all* $b \in \{2,4,6,...\}$ *and for some* $c \in \{1,3,5,...\}$ *and for some* $\mathbf{p} > 2$ *with* $\gcd(a,b) = 1$*:*

$$\left\{\left[\frac{a^n + b^n}{a+b} = c \wedge \ \gcd(a,(a+b)c) = \gcd(b,(a+b)c) = 1\right] \Rightarrow \right.$$

$$(a(a+b)c)^n + (b(a+b)c)^n = ((a+b)c)^{n+1} = C^{n+1} = A^n + B^n \Rightarrow$$

$$\left.\mathbf{p} \mid (a+b)c, \ C, \ A, \ B\right\}.$$

This is the lemma 2.

Example 4.

$$1^3 + 2^3 = 3(1+2) = 3^2 \Rightarrow 9^3 + 18^3 = 3^8 = 9^4 \Rightarrow \left(3^3 + 6^3 = 3^{3+2} \text{ [3]}\right).$$

This is the example 4.

Lemma 3. *For all* $n \in \{3,5,7,...\}$ *and for all* $a, \ b \in \{1,3,5,...\}$ *and for some* $c \in \{1,3,5,...\}$ *and for some* $\mathbf{p} \geq 2$ *with* $\gcd(a,b) = 1$*:*

$$\left\{\left[\frac{a^n + b^n}{a+b} = c \wedge \ \gcd(a,(a+b)c) = \gcd(b,(a+b)c) = 1\right] \Rightarrow \right.$$

$$(a(a+b)c)^n + (b(a+b)c)^n = ((a+b)c)^{n+1} = C^{n+1} = A^n + B^n \Rightarrow$$

$$\left.\mathbf{p} \mid (a+b)c, \ C, \ A, \ B\right\}.$$

This is the lemma 3.

Example 5.

$$1^n + 1^n = 2 \Rightarrow \left(2^n + 2^n = 2^{n+1} \text{ [3]}\right).$$

This is the example 5.

Lemma 4. *For all* $n \in \{3,5,7,...\}$ *and for all* $a, \ b \in \{1,3,5,...\}$ *and for some* $c \in \{1,3,5,...\}$ *and for some* $\mathbf{p} \geq 2$ *with* $a > b$ *and* $\gcd(a,b) = 1$*:*

$$\left\{\left[\frac{a^n - b^n}{a-b} = c \wedge \ \gcd(a,(a-b)c) = \gcd(b,(a-b)c) = 1\right] \Rightarrow \right.$$

$$(a(a-b)c)^n - (b(a-b)c)^n = ((a-b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\left.\mathbf{p} \mid (a-b)c, \ A, \ C, \ B\right\}.$$

This is the lemma 4.

Lemma 5. *For all* $n, \ a \in \{3,5,7,...\}$ *and for all* $b \in \{2,4,6,...\}$ *and for some* $c \in \{1,3,5,...\}$ *and for some* $\mathbf{p} > 2$ *with* $a > b$ *and* $\gcd(a,b) = 1$*:*

$$\left\{\left[\frac{a^n - b^n}{a-b} = c \wedge \ \gcd(a,(a-b)c) = \gcd(b,(a-b)c) = 1\right] \Rightarrow \right.$$

$$(a(a-b)c)^n - (b(a-b)c)^n = ((a-b)c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\left.\mathbf{p} \mid (a-b)c, \ A, \ C, \ B\right\}.$$

This is the lemma 5.

Lemma 6. *For all* $n \in \{4, 6, 8, ...\}$ *and for all* $a,\ b \in \{1, 3, 5, ...\}$ *and for some* $c \in \{2, 4, 6, ...\}$ *and for some* $\mathbf{p} \geq 2$ *with* $a > b$ *and* $\gcd(a, b) = 1$*:*

$$\left\{ \left[\frac{a^n - b^n}{a - b} = c \wedge \ \gcd(a, (a - b)\, c) = \gcd(b, (a - b)\, c) = 1 \right] \Rightarrow \right.$$

$$(a\,(a - b)\, c)^n - (b\,(a - b)\, c)^n = ((a - b)\, c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\left. \mathbf{p} \mid (a - b)\, c,\ A,\ C,\ B \right\}.$$

This is the lemma 6.

Lemma 7. *For all* $n \in \{4, 6, 8, ...\}$ *and for all* $a \in \{2, 4, 6, ...\}$ *and for all* $b \in \{1, 3, 5, ...\}$ *and for some* $c \in \{1, 3, 5, ...\}$ *and for some* $\mathbf{p} > 2$ *with* $a > b$ *and* $\gcd(a, b) = 1$*:*

$$\left\{ \left[\frac{a^n - b^n}{a - b} = c \wedge \ \gcd(a, (a - b)\, c) = \gcd(b, (a - b)\, c) = 1 \right] \Rightarrow \right.$$

$$(a\,(a - b)\, c)^n - (b\,(a - b)\, c)^n = ((a - b)\, c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\left. \mathbf{p} \mid (a - b)\, c,\ A,\ C,\ B \right\}.$$

This is the lemma 7.

Corollary 3. *For all* $n \in \mathbb{N}_3$ *and for all* $a \in \mathbb{N}_2$ *and for some* $c \in \mathbb{N}_1$*:*

$$\left\{ [a\,(a^n - 1)]^n - (a^n - 1)^n = (a^n - 1)^{n+1} = \right.$$

$$[a\,(a - 1)\, c]^n - [(a - 1)\, c]^n = [(a - 1)\, c]^{n+1} \Rightarrow$$

$$\left. \left[(a^n - 1)^{2n} + (a^n - 1)^{2n+1} = \left[a\,(a^n - 1)^2 \right]^n \ \text{[3]} \right] \right\}.$$

This is the corollary 3.

Lemma 8. *For all* $n \in \{4, 6, 8, ...\}$ *and for all* $a \in \{3, 5, 7, ...\}$ *and for all* $b \in \{2, 4, 6, ...\}$ *and for some* $c \in \{1, 3, 5, ...\}$ *and for some* $\mathbf{p} > 2$ *with* $a > b$ *and* $\gcd(a, b) = 1$*:*

$$\left\{ \left[\frac{a^n - b^n}{a - b} = c \wedge \ \gcd(a, (a - b)\, c) = \gcd(b, (a - b)\, c) = 1 \right] \Rightarrow \right.$$

$$(a\,(a - b)\, c)^n - (b\,(a - b)\, c)^n = ((a - b)\, c)^{n+1} = A^{n+1} = C^n - B^n \Rightarrow$$

$$\left. \mathbf{p} \mid (a - b)\, c,\ A,\ C,\ B \right\}.$$

This is the lemma 8.

Lemma 9. *For all* $n \in \{4, 6, 8, ...\}$ *and for all* $a \in \{1, 3, 5, ...\}$ *and for all* $b \in \{2, 4, 6, ...\}$ *and for some* $c \in \{1, 3, 5, ...\}$ *and for some* $\mathbf{p} > 2$ *with* $\gcd(a, b) = 1$*:*

$$\{ [a^n + b^n = c \wedge \ \gcd(a, c) = \gcd(b, c) = 1] \Rightarrow$$

$$(ac)^n + (bc)^n = c^{n+1} = C^{n+1} = A^n + B^n \Rightarrow \mathbf{p} \mid a^n + b^n,\ C,\ A,\ B \}.$$

This is the lemma 9.

Lemma 10. *For all* $n\in\{4,6,8,...\}$ *and for all* $a,\,b\in\{1,3,5,...\}$ *and for some* $c\in\{1,3,5,...\}$ *and for some* $\mathbf{p}\geq 2$ *with* $\gcd(a,b)=1$*:*

$$\{[a^n+b^n=2c\wedge\ \gcd(a,2c)=\gcd(b,2c)=1]\Rightarrow$$

$$\left.(2ac)^n+(2bc)^n=(2c)^{n+1}=C^{n+1}=A^n+B^n\Rightarrow\mathbf{p}\mid 2c,\ C,\ A,\ B\right\}.$$

This is the lemma 10.

Proof of Andy Beal Conjecture. Each from the above lemmas gives **always** one of the below equations

$$\left(A^{n+1}+B^n=C^n\vee\ A^n+B^n=C^{n+1}\right),$$

where $A,\,B$, and C have the common prime factor $\mathbf{p}\geq 2$.

The above lemmans gives all solutions of the equation $A^x+B^y=C^z$ such that $A,\,B$, and C have the common prime factor $\mathbf{p}\geq 2$. Moreover the number of the solutions $[A,B,C]$ is infinite.

For some $n,\,x,\,y,\,z\in\mathbb{N}_3$ and for some coprime $A,\,B,\,C\in\mathbb{N}_1$:

$$\left[\left(A^{n+1}+B^n=C^n\vee\ A^n+B^n=C^{n+1}\right)\equiv A^x+B^y=C^z\right].$$

Therefore for some $n,\,x,\,y,\,z\in\mathbb{N}_3$ and for all $d\in\mathbb{N}_2$ and for some coprime numbers $A,\,B,\,C\in\mathbb{N}_1$ and for some coprime numbers $a,\,b,\,c\in\mathbb{N}_1$:

$$\{[[A\,(dA)^n+(dB)^n=(dC)^n\equiv 0]\ \vee\ [(dA)^n+(dB)^n=C\,(dC)^n\equiv 0]]\ \equiv$$

$$\left[\left[(dA)^x+\left(dB^{\frac{y}{x}}\right)^x=\left(dC^{\frac{z}{x}}\right)^x\wedge\ x\mid y,\ z\wedge\ x\neq y\neq z\neq x\right]\ \vee\right.$$

$$[(dA)^x+(db^y)^x=(dc^z)^x\wedge\ x\nmid y,\ z\wedge\ y\neq z\wedge\ b^x=B\wedge\ c^x=C]\ \vee$$

$$\left[\left(dA^{\frac{x}{y}}\right)^y+(dB)^y=\left(dC^{\frac{z}{y}}\right)^y\wedge\ y\mid x,\ z\wedge\ x\neq y\neq z\neq x\right]\ \vee$$

$$[(da^x)^y+(dB)^y=(dc^z)^y\wedge\ y\nmid x,\ z\wedge\ x\neq z\wedge\ a^y=A\wedge\ c^y=C]\ \vee$$

$$\left[\left(dA^{\frac{x}{z}}\right)^z+\left(dB^{\frac{y}{z}}\right)^z=(dC)^z\wedge\ z\mid x,\ y\wedge\ x\neq y\neq z\neq x\right]\ \vee$$

$$[(da^x)^z+(db^y)^z=(dC)^z\wedge\ z\nmid x,\ y\wedge\ x\neq y\wedge\ a^z=A\wedge\ b^z=B]\ \vee$$

$$\left[(dA)^x+(dB)^x=\left(dC^{\frac{z}{x}}\right)^x\wedge\ x\mid z\wedge\ x=y\wedge\ x\neq z\right]\ \vee$$

$$[(dA)^x+(dB)^x=(dc^z)^x\wedge\ x\nmid z\wedge\ x=y\wedge\ c^x=C]\ \vee$$

$$\left[(dA)^x+\left(dB^{\frac{y}{x}}\right)^x=(dC)^x\wedge\ x\mid y\wedge\ x=z\wedge\ x\neq y\right]\ \vee$$

$$[(dA)^x+(db^y)^x=(dC)^x\wedge\ x\nmid y\wedge\ x=z\wedge\ b^x=B]\ \vee$$

$$\left[\left(dA^{\frac{x}{y}}\right)^y+(dB)^y=(dC)^y\wedge\ y\mid x\wedge\ y=z\wedge\ x\neq y\right]\ \vee$$

$$[(da^x)^y+(dB)^y=(dC)^y\wedge\ y\nmid x\wedge\ y=z\wedge\ a^y=A]\ \vee$$

$$[(dA)^x+(dB)^x=(dC)^x\wedge\ x=y=z]]\},$$

which is false also in view of the above proof of FLT. This is the proof. ☐

References

[1] http://www.bealconjecture.com/

[2] Dolan, S. : Fermat's method of descente infinie, Mathematical Gazette 95, July 2011, 269-271.

[3] http://en.wikipedia.org/wiki/Beal's_conjecture

[4] Gładki, P.: http://www.math.us.edu.pl/~pgladki/faq/node135.html

[5] http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf

[6] Mazur. B. : About The Cover: Diohantus's Arithmetica, http://www.ams.org/journals/bull/2006-43-03/S0273-0979-06-01123-2/S0273-0979-06-01123-2.pdf

[7] Weil, A. : Fermat's Only Published Proof, http://www.cut-the-knot.org/arithmetic/FermatsInfiniteDescent.shtml

[8] http://en.wikipedia.org/wiki/Fermat's_Last_Theorem

Lublin-POLAND
E-mail address: lwgula@wp.pl