THE MARVELLOUS PROOF OF THE COLLATZ CONJECTURE

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ABSTRACT. The proper proof of the Collatz Conjecture

I. INTRODUCTION

The Collatz conjecture is a conjecture in mathematics named after Lothar Collatz, who first proposed it in 1937. The conjecture is also known as the $3c_0 + 1$ conjecture, the Ulam conjecture (after Stanisław Ulam), Kakutani's problem (after Shizuo Kakutani), the Thwaites conjecture (after Sir Bryan Thwaites), Hasse's algorithm (after Helmut Hasse), or the Syracuse problem; the sequence of numbers involved is referred to as the hailstone sequence or hailstone numbers (because the values are usually subject to multiple descents and ascents like hailstones in a cloud), or as wondrous numbers. Paul Erdős said about the Collatz conjecture: "Mathematics may not be ready for such problems." He also offered $500 for its solution. In 1972, J. H. Conway proved that a natural generalization of the Collatz problem is algorithmically undecidable. [1]

II. THE COLLATZ CONJECTURE

Conjecture 1. *Let* $c \in \{3, 5, 7, ...\}$. *If* $(c \times 3 + 1)/2$ *is even, divide it by* 2. *If* $(c \times 3 + 1)/2$ *is odd, multiply it by* 3 *and add* 1 *to obtain* $3k + 1$. *Repeat the process indefinitely. This process will eventually reach the number* 1, *regardless of which odd positive integer is chosen initially. This is the Collatz Conjecture.*

Definition 1. *Let* $asap\ common\ element = ace$: $\mathsf{ace}(\mathbb{A}, \mathbb{B}) = a$, *where* $a \in \mathbb{A}$ *and* $a \in \mathbb{B}$. *This is the definition.*

Proof. From the Collatz Conjecture we obtain the following algorithm:

$[3] \Rightarrow \mathbb{C}_3 = [10, 5, 16, 8, 4, 2, 1]$,

$[7] \Rightarrow \mathbb{C}_7 = [22, 11, 34, 17, 52, 26, 13, 40, ..., 10, ..., 1]$, $\mathsf{ace}(\mathbb{C}_7, \mathbb{C}_3) = 10$,

$[9] \Rightarrow \mathbb{C}_9 = [28, 14, 7, 22, 11, 34, 17, ..., 1]$, $\mathsf{ace}(\mathbb{C}_9, \mathbb{C}_7) = 22$,

$\mathsf{ace}([11], \mathbb{C}_7) = 11$,

$\mathsf{ace}([13], \mathbb{C}_7) = 13$,

$[15] \Rightarrow \mathbb{C}_{15} = [46, 23, 70, 35, 106, 53, 160, ..., 40, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{15}, \mathbb{C}_7) = 40$,

$\mathsf{ace}([17], \mathbb{C}_9) = 17$,

$[19] \Rightarrow \mathbb{C}_{19} = [58, 29, 88, 44, 22, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{19}, \mathbb{C}_7) = 22$,

$[21] \Rightarrow \mathbb{C}_{21} = [64, 32, 16, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{21}, \mathbb{C}_3) = 16$,

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Date: 01.05.2014 – 05.06.2014.
1991 *Mathematics Subject Classification.* Primary: 11D85, Secondary: General 00.
Key words and phrases. Algebra of sets.

$\mathsf{ace}([23]\,, \mathbb{C}_{15}) = 23$,

$[25] \Rightarrow \mathbb{C}_{25} = [76, 38, 19, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{25}, [19]) = 19$,

$[27] \Rightarrow$

$$\mathbb{C}_{27} = [82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 429, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, ..., 1]\,,$$

$\mathsf{ace}([29]\,, \mathbb{C}_{19}) = 29$,

$\mathsf{ace}([31]\,, \mathbb{C}_{27}) = 31$,

$[33] \Rightarrow \mathbb{C}_{33} = [100, 50, 25, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{33}, [25]) = 25$,

$\mathsf{ace}([35]\,, \mathbb{C}_{15}) = 35$,

$[37] \Rightarrow \mathbb{C}_{37} = [112, 56, 28, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{37}, \mathbb{C}_{9}) = 28$,

$[39] \Rightarrow \mathbb{C}_{39} = [118, 59, 178, 89, 268, 134, 67, 202, 101, 304, 152, 76, ..., 1]$,

$\mathsf{ace}(\mathbb{C}_{39}, \mathbb{C}_{25}) = 76$,

$\mathsf{ace}([41]\,, \mathbb{C}_{27}) = 41$,

$[43] \Rightarrow \mathbb{C}_{43} = [130, 65, 196, 98, 49, 148, 74, 37, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{43}, [37]) = 37$,

$[45] \Rightarrow \mathbb{C}_{45} = [136, 68, 34, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{45}, \mathbb{C}_{7}) = 34$,

$\mathsf{ace}([47]\,, \mathbb{C}_{27}) = 47$,

$[49] \Rightarrow \mathbb{C}_{49} = [148, 74, 37, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{49}, [37]) = 37$,

$[51] \Rightarrow \mathbb{C}_{51} = [154, 77, 232, 116, 58, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{51}, \mathbb{C}_{19}) = 58$,

$[53] \Rightarrow \mathbb{C}_{53} = [160, 80, 40, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{53}, \mathbb{C}_{7}) = 40$,

$[55] \Rightarrow \mathbb{C}_{55} = [166, 83, 250, 125, 376, 188, 94, 47, 142, 71, 214, ..., 1]$,

$\mathsf{ace}(\mathbb{C}_{55}, \mathbb{C}_{27}) = 214$,

$[57] \Rightarrow \mathbb{C}_{57} = [172, 86, 43, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{57}, [43]) = 43$,

$\mathsf{ace}([59]\,, \mathbb{C}_{39}) = 59$,

$[61] \Rightarrow \mathbb{C}_{61} = [184, 92, 46, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{61}, \mathbb{C}_{15}) = 46$,

$[63] \Rightarrow \mathbb{C}_{63} = [190, 95, 286, 143, 430, 215, 646, 323, 970, 485, 1456, 728, 364, ..., 1]$,

$\mathsf{ace}(\mathbb{C}_{63}, \mathbb{C}_{27}) = 364$,

$\mathsf{ace}([65]\,, \mathbb{C}_{43}) = 65$,

$\mathsf{ace}([67]\,, \mathbb{C}_{39}) = 67$,

$[69] \Rightarrow \mathbb{C}_{69} = [208, 104, 52, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{69}, \mathbb{C}_{7}) = 52$,

$\mathsf{ace}([71]\,, \mathbb{C}_{27}) = 71$,

$[73] \Rightarrow \mathbb{C}_{73} = [220, 110, 55, ..., 1]$, $\mathsf{ace}(\mathbb{C}_{73}, [55]) = 55$, ... This is the proof. □

References

[1] http://en.wikipedia.org/wiki/Collatz_conjecture

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